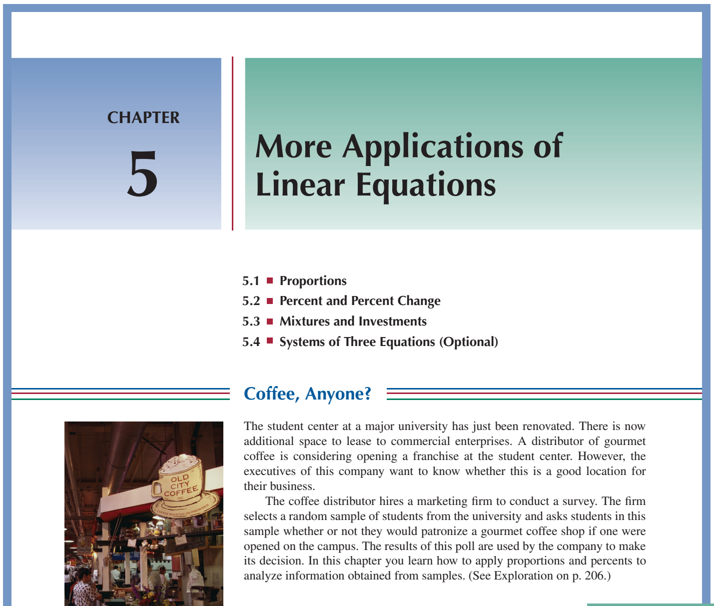
# Chapter Opening Features

## Chapter Opener

Each chapter opens with a list of the objectives covered in the chapter, an application linking chapter topics to a real world example, and a photo to enhance the application. The sections are listed clearly and serve as an organizational tool for the chapter. The same section titles appear throughout the chapter, along with objectives listed below them. Students are able to use these objectives as a guide for what they should learn in each section.



CHAPTER

5

More Applications of Linear Equations

5.1 ■ Proportions
5.2 ■ Percent and Percent Change
5.3 ■ Mixtures and Investments
5.4 ■ Systems of Three Equations (Optional)

**Coffee, Anyone?**

The student center at a major university has just been renovated. There is now additional space to lease to commercial enterprises. A distributor of gourmet coffee is considering opening a franchise at the student center. However, the executives of this company want to know whether this is a good location for their business.

The coffee distributor hires a marketing firm to conduct a survey. The firm selects a random sample of students from the university and asks students in this sample whether or not they would patronize a gourmet coffee shop if one were opened on the campus. The results of this poll are used by the company to make its decision. In this chapter you learn how to apply proportions and percents to analyze information obtained from samples. (See Exploration on p. 206.)

Need help? For on-line resources, visit this web site: **math.college.hmco.c**

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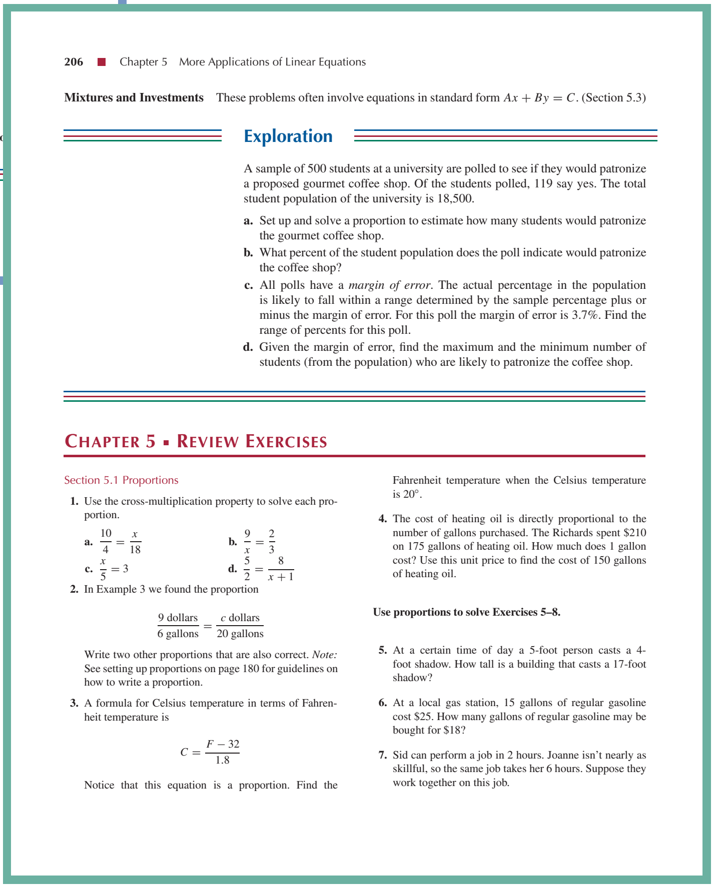
## Exploration

Explorations at the end of the chapter give students the opportunity to work problems and answer questions related to the application introduced in the Chapter Opener. By using the strategies they were taught within the chapter, students are able to complete the Explorations and connect the mathematics to real world experiences.



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**Mixtures and Investments** These problems often involve equations in standard form $Ax + By = C$. (Section 5.3)

**Exploration**

A sample of 500 students at a university are polled to see if they would patronize a proposed gourmet coffee shop. Of the students polled, 119 say yes. The total student population of the university is 18,500.

**a.** Set up and solve a proportion to estimate how many students would patronize the gourmet coffee shop.

**b.** What percent of the student population does the poll indicate would patronize the coffee shop?

**c.** All polls have a *margin of error*. The actual percentage in the population is likely to fall within a range determined by the sample percentage plus or minus the margin of error. For this poll the margin of error is 3.7%. Find the range of percents for this poll.

**d.** Given the margin of error, find the maximum and the minimum number of students (from the population) who are likely to patronize the coffee shop.

**CHAPTER 5 ▪ REVIEW EXERCISES**

Section 5.1 Proportions

**1.** Use the cross-multiplication property to solve each proportion.

**a.** $\frac{10}{4} = \frac{x}{18}$ **b.** $\frac{9}{x} = \frac{2}{3}$

**c.** $\frac{x}{5} = 3$ **d.** $\frac{5}{2} = \frac{8}{x+1}$

**2.** In Example 3 we found the proportion

$$\frac{9 \text{ dollars}}{6 \text{ gallons}} = \frac{c \text{ dollars}}{20 \text{ gallons}}$$

Write two other proportions that are also correct. *Note:* See setting up proportions on page 180 for guidelines on how to write a proportion.

**3.** A formula for Celsius temperature in terms of Fahrenheit temperature is

$$C = \frac{F - 32}{1.8}$$

Notice that this equation is a proportion. Find the Fahrenheit temperature when the Celsius temperature is 20°.

**4.** The cost of heating oil is directly proportional to the number of gallons purchased. The Richards spent \$210 on 175 gallons of heating oil. How much does 1 gallon cost? Use this unit price to find the cost of 150 gallons of heating oil.

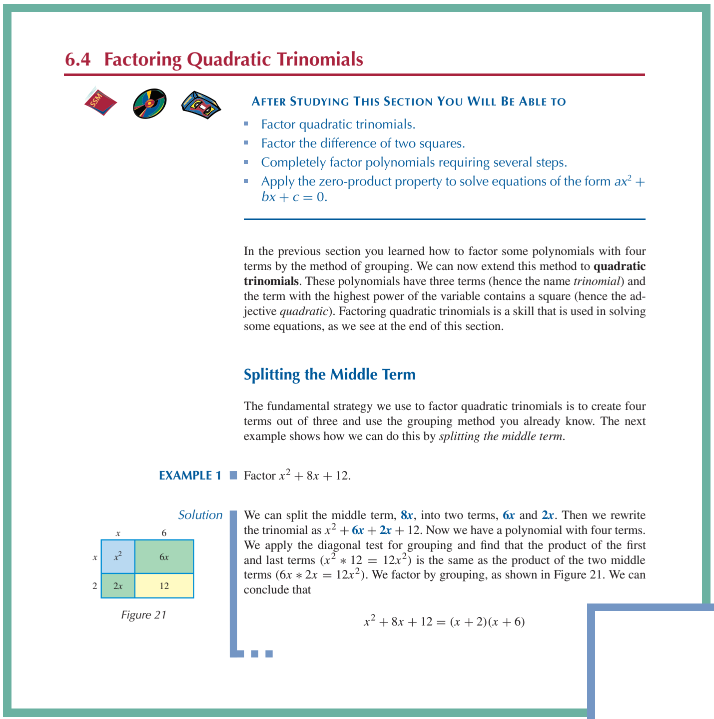
**Use proportions to solve Exercises 5–8.**

**5.** At a certain time of day a 5-foot person casts a 4-foot shadow. How tall is a building that casts a 17-foot shadow?

**6.** At a local gas station, 15 gallons of regular gasoline cost \$25. How many gallons of regular gasoline may be bought for \$18?

**7.** Sid can perform a job in 2 hours. Joanne isn't nearly as skillful, so the same job takes her 6 hours. Suppose they work together on this job.

# Student Pedagogy



## 6.4 Factoring Quadratic Trinomials

AFTER STUDYING THIS SECTION YOU WILL BE ABLE TO

- Factor quadratic trinomials.
- Factor the difference of two squares.
- Completely factor polynomials requiring several steps.
- Apply the zero-product property to solve equations of the form $ax^2 + bx + c = 0$.

In the previous section you learned how to factor some polynomials with four terms by the method of grouping. We can now extend this method to **quadratic trinomials**. These polynomials have three terms (hence the name *trinomial*) and the term with the highest power of the variable contains a square (hence the adjective *quadratic*). Factoring quadratic trinomials is a skill that is used in solving some equations, as we see at the end of this section.

### Splitting the Middle Term

The fundamental strategy we use to factor quadratic trinomials is to create four terms out of three and use the grouping method you already know. The next example shows how we can do this by *splitting the middle term.*

**EXAMPLE 1** ■ Factor $x^2 + 8x + 12$.

*Solution* We can split the middle term, $\mathbf{8x}$, into two terms, $\mathbf{6x}$ and $\mathbf{2x}$. Then we rewrite the trinomial as $x^2 + \mathbf{6x} + \mathbf{2x} + 12$. Now we have a polynomial with four terms. We apply the diagonal test for grouping and find that the product of the first and last terms $(x^2 * 12 = 12x^2)$ is the same as the product of the two middle terms $(6x * 2x = 12x^2)$. We factor by grouping, as shown in Figure 21. We can conclude that

$$x^2 + 8x + 12 = (x + 2)(x + 6)$$

|  | $x$ | $6$ |
|---|---|---|
| $x$ | $x^2$ | $6x$ |
| $2$ | $2x$ | $12$ |

*Figure 21*

## Icons

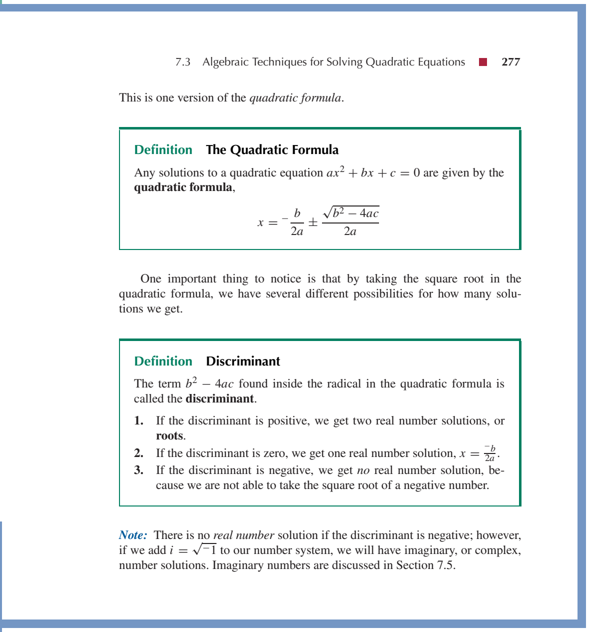
for the Student Solutions Manual, Student CD, and Videos appear at the beginning of every section. These make students aware of the additional study resources that are available for each section.

## Key Terms

Bolded and italicized terms appear throughout the chapters to highlight key points. The bolded terms are also reviewed in the Key Concepts section at the end of each chapter.

## Definition and Property Boxes

are incorporated into each chapter to emphasize important terms and formulas.



7.3 Algebraic Techniques for Solving Quadratic Equations ■ 277

This is one version of the *quadratic formula*.

**Definition** **The Quadratic Formula**

Any solutions to a quadratic equation $ax^2 + bx + c = 0$ are given by the **quadratic formula**,

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

One important thing to notice is that by taking the square root in the quadratic formula, we have several different possibilities for how many solutions we get.

**Definition** **Discriminant**

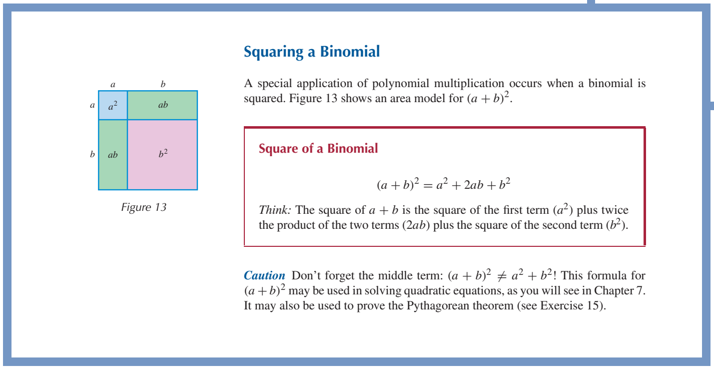
The term $b^2 - 4ac$ found inside the radical in the quadratic formula is called the **discriminant**.

1. If the discriminant is positive, we get two real number solutions, or **roots**.
2. If the discriminant is zero, we get one real number solution, $x = \frac{-b}{2a}$.
3. If the discriminant is negative, we get *no* real number solution, because we are not able to take the square root of a negative number.

***Note:*** There is no *real number* solution if the discriminant is negative; however, if we add $i = \sqrt{-1}$ to our number system, we will have imaginary, or complex, number solutions. Imaginary numbers are discussed in Section 7.5.

## Notes and Cautions

appear, as needed, to keep students focused and to make them aware of possible common errors.



### Squaring a Binomial

A special application of polynomial multiplication occurs when a binomial is squared. Figure 13 shows an area model for $(a + b)^2$.

|  | $a$ | $b$ |
|---|---|---|
| $a$ | $a^2$ | $ab$ |
| $b$ | $ab$ | $b^2$ |

*Figure 13*

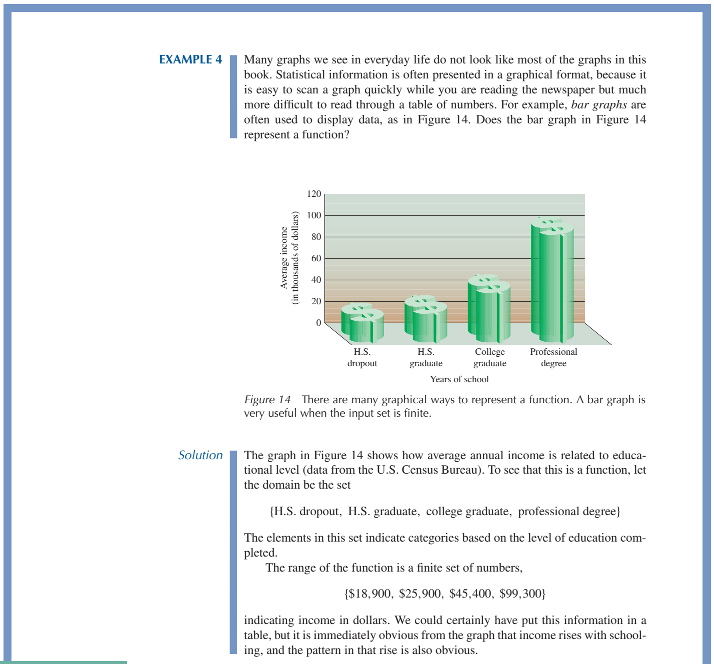
**Square of a Binomial**

$$(a + b)^2 = a^2 + 2ab + b^2$$

*Think:* The square of $a + b$ is the square of the first term $(a^2)$ plus twice the product of the two terms $(2ab)$ plus the square of the second term $(b^2)$.

***Caution*** Don't forget the middle term: $(a + b)^2 \neq a^2 + b^2$! This formula for $(a + b)^2$ may be used in solving quadratic equations, as you will see in Chapter 7. It may also be used to prove the Pythagorean theorem (see Exercise 15).

# Applications



**EXAMPLE 4** Many graphs we see in everyday life do not look like most of the graphs in this book. Statistical information is often presented in a graphical format, because it is easy to scan a graph quickly while you are reading the newspaper but much more difficult to read through a table of numbers. For example, *bar graphs* are often used to display data, as in Figure 14. Does the bar graph in Figure 14 represent a function?

*Figure 14* There are many graphical ways to represent a function. A bar graph is very useful when the input set is finite.

*Solution* The graph in Figure 14 shows how average annual income is related to educational level (data from the U.S. Census Bureau). To see that this is a function, let the domain be the set

{H.S. dropout, H.S. graduate, college graduate, professional degree}

The elements in this set indicate categories based on the level of education completed.

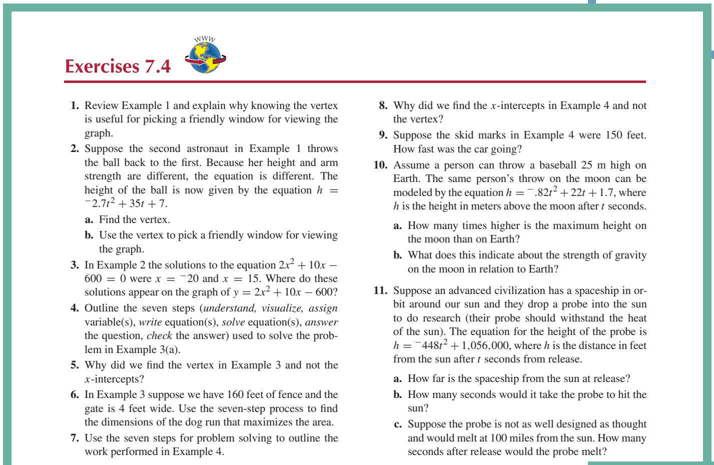
The range of the function is a finite set of numbers,

{\$18,900, \$25,900, \$45,400, \$99,300}

indicating income in dollars. We could certainly have put this information in a table, but it is immediately obvious from the graph that income rises with schooling, and the pattern in that rise is also obvious.

## Example and Solution

Applications are used throughout the material to enhance understanding of the mathematics. Examples of realistic situations are presented, followed by step-by-step solutions.



### Exercises 7.4

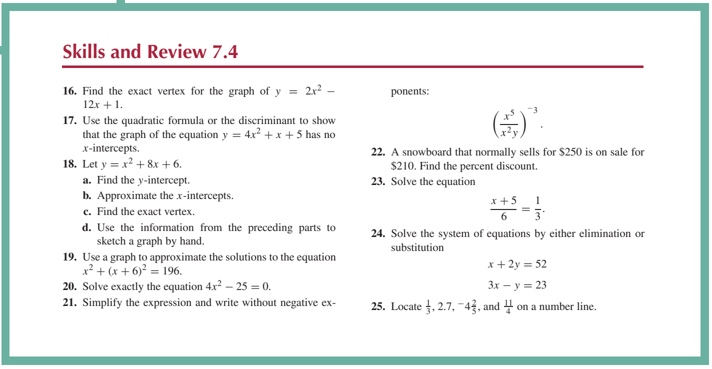
1. Review Example 1 and explain why knowing the vertex is useful for picking a friendly window for viewing the graph.
2. Suppose the second astronaut in Example 1 throws the ball back to the first. Because her height and arm strength are different, the equation is different. The height of the ball is now given by the equation $h = {}^{-}2.7t^2 + 35t + 7$.
   a. Find the vertex.
   b. Use the vertex to pick a friendly window for viewing the graph.
3. In Example 2 the solutions to the equation $2x^2 + 10x - 600 = 0$ were $x = {}^{-}20$ and $x = 15$. Where do these solutions appear on the graph of $y = 2x^2 + 10x - 600$?
4. Outline the seven steps (*understand, visualize, assign* variable(s), *write* equation(s), *solve* equation(s), *answer* the question, *check* the answer) used to solve the problem in Example 3(a).
5. Why did we find the vertex in Example 3 and not the $x$-intercepts?
6. In Example 3 suppose we have 160 feet of fence and the gate is 4 feet wide. Use the seven-step process to find the dimensions of the dog run that maximizes the area.
7. Use the seven steps for problem solving to outline the work performed in Example 4.
8. Why did we find the $x$-intercepts in Example 4 and not the vertex?
9. Suppose the skid marks in Example 4 were 150 feet. How fast was the car going?
10. Assume a person can throw a baseball 25 m high on Earth. The same person's throw on the moon can be modeled by the equation $h = {}^{-}.82t^2 + 22t + 1.7$, where $h$ is the height in meters above the moon after $t$ seconds.
    a. How many times higher is the maximum height on the moon than on Earth?
    b. What does this indicate about the strength of gravity on the moon in relation to Earth?
11. Suppose an advanced civilization has a spaceship in orbit around our sun and they drop a probe into the sun to do research (their probe should withstand the heat of the sun). The equation for the height of the probe is $h = {}^{-}448t^2 + 1{,}056{,}000$, where $h$ is the distance in feet from the sun after $t$ seconds from release.
    a. How far is the spaceship from the sun at release?
    b. How many seconds would it take the probe to hit the sun?
    c. Suppose the probe is not as well designed as thought and would melt at 100 miles from the sun. How many seconds after release would the probe melt?

## Section Exercises

Each section ends with two types of problems.

## Exercises and Skills and Review

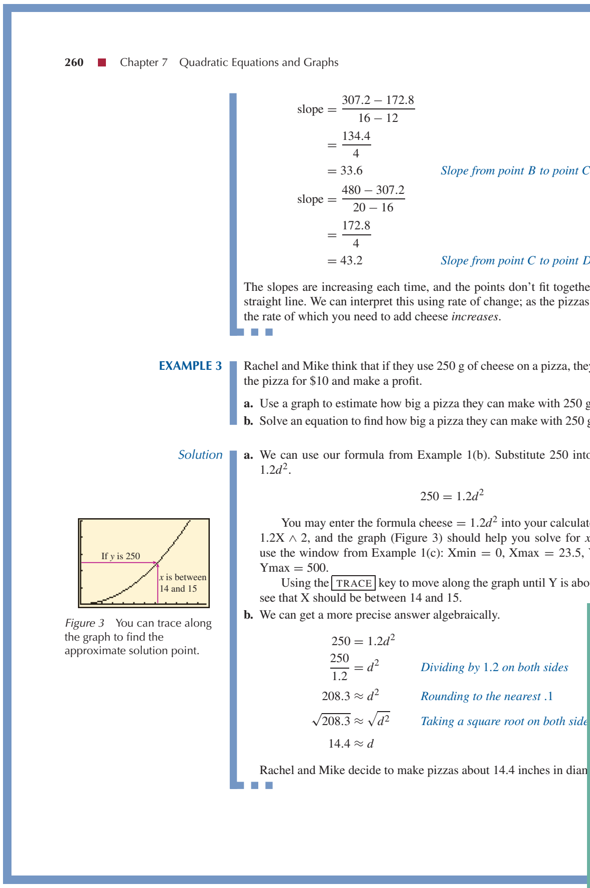
reinforce what students have learned. Exercises are comprehensive, manageable in length, and intended to be assigned in full.



### Skills and Review 7.4

16. Find the exact vertex for the graph of $y = 2x^2 - 12x + 1$.
17. Use the quadratic formula or the discriminant to show that the graph of the equation $y = 4x^2 + x + 5$ has no $x$-intercepts.
18. Let $y = x^2 + 8x + 6$.
    a. Find the $y$-intercept.
    b. Approximate the $x$-intercepts.
    c. Find the exact vertex.
    d. Use the information from the preceding parts to sketch a graph by hand.
19. Use a graph to approximate the solutions to the equation $x^2 + (x + 6)^2 = 196$.
20. Solve exactly the equation $4x^2 - 25 = 0$.
21. Simplify the expression and write without negative exponents:
    $$\left(\frac{x^5}{x^2 y}\right)^{^{-}3}.$$
22. A snowboard that normally sells for \$250 is on sale for \$210. Find the percent discount.
23. Solve the equation
    $$\frac{x+5}{6} = \frac{1}{3}.$$
24. Solve the system of equations by either elimination or substitution
    $$x + 2y = 52$$
    $$3x - y = 23$$
25. Locate $\frac{1}{3}$, 2.7, ${}^{-}4\frac{2}{5}$, and $\frac{11}{4}$ on a number line.

# Graphing Calculator



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$$\text{slope} = \frac{307.2 - 172.8}{16 - 12}$$
$$= \frac{134.4}{4}$$
$$= 33.6 \qquad \textit{Slope from point B to point C}$$
$$\text{slope} = \frac{480 - 307.2}{20 - 16}$$
$$= \frac{172.8}{4}$$
$$= 43.2 \qquad \textit{Slope from point C to point D}$$

The slopes are increasing each time, and the points don't fit together to form a straight line. We can interpret this using rate of change; as the pizzas get bigger, the rate of which you need to add cheese *increases*.

**EXAMPLE 3** Rachel and Mike think that if they use 250 g of cheese on a pizza, they could sell the pizza for $10 and make a profit.

**a.** Use a graph to estimate how big a pizza they can make with 250 g of cheese.
**b.** Solve an equation to find how big a pizza they can make with 250 g of cheese.

*Solution* **a.** We can use our formula from Example 1(b). Substitute 250 into cheese $= 1.2d^2$.

$$250 = 1.2d^2$$

You may enter the formula cheese $= 1.2d^2$ into your calculator as Y1 = 1.2X ∧ 2, and the graph (Figure 3) should help you solve for $x$. You may use the window from Example 1(c): Xmin = 0, Xmax = 23.5, Ymin = 0, Ymax = 500.

Using the TRACE key to move along the graph until Y is about 250, you see that X should be between 14 and 15.

If y is 250

x is between 14 and 15

*Figure 3* You can trace along the graph to find the approximate solution point.

**b.** We can get a more precise answer algebraically.

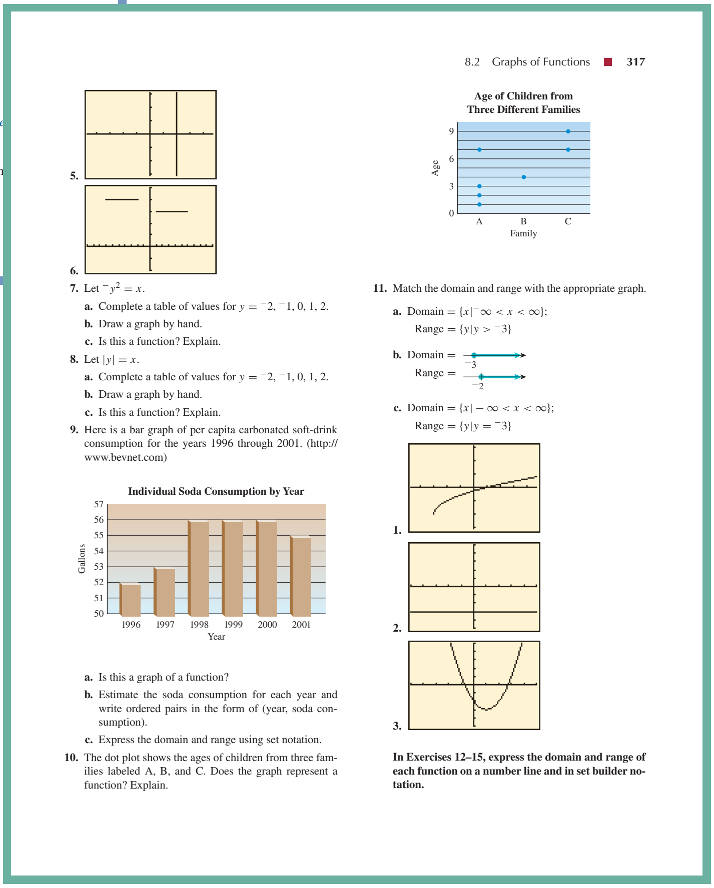
$$250 = 1.2d^2$$
$$\frac{250}{1.2} = d^2 \qquad \textit{Dividing by 1.2 on both sides}$$
$$208.3 \approx d^2 \qquad \textit{Rounding to the nearest .1}$$
$$\sqrt{208.3} \approx \sqrt{d^2} \qquad \textit{Taking a square root on both side}$$
$$14.4 \approx d$$

Rachel and Mike decide to make pizzas about 14.4 inches in dian

## Graphing Calculator

examples are incorporated throughout the chapters to guide students through problems. Screens are shown directly on the page to provide a visual example. These also help students follow along step-by-step on their own graphing calculator.

The graphing theme is carried throughout section and chapter-end material.



8.2 Graphs of Functions ■ 317

5.

6.

**7.** Let $^-y^2 = x$.

**a.** Complete a table of values for $y = {}^-2, {}^-1, 0, 1, 2$.
**b.** Draw a graph by hand.
**c.** Is this a function? Explain.

**8.** Let $|y| = x$.

**a.** Complete a table of values for $y = {}^-2, {}^-1, 0, 1, 2$.
**b.** Draw a graph by hand.
**c.** Is this a function? Explain.

**9.** Here is a bar graph of per capita carbonated soft-drink consumption for the years 1996 through 2001. (http://www.bevnet.com)

**Individual Soda Consumption by Year**

**a.** Is this a graph of a function?
**b.** Estimate the soda consumption for each year and write ordered pairs in the form of (year, soda consumption).
**c.** Express the domain and range using set notation.

**10.** The dot plot shows the ages of children from three families labeled A, B, and C. Does the graph represent a function? Explain.

**Age of Children from Three Different Families**

**11.** Match the domain and range with the appropriate graph.

**a.** Domain $= \{x|{}^-\infty < x < \infty\}$;
Range $= \{y|y > {}^-3\}$

**b.** Domain =
Range =

**c.** Domain $= \{x| - \infty < x < \infty\}$;
Range $= \{y|y = {}^-3\}$

1.

2.

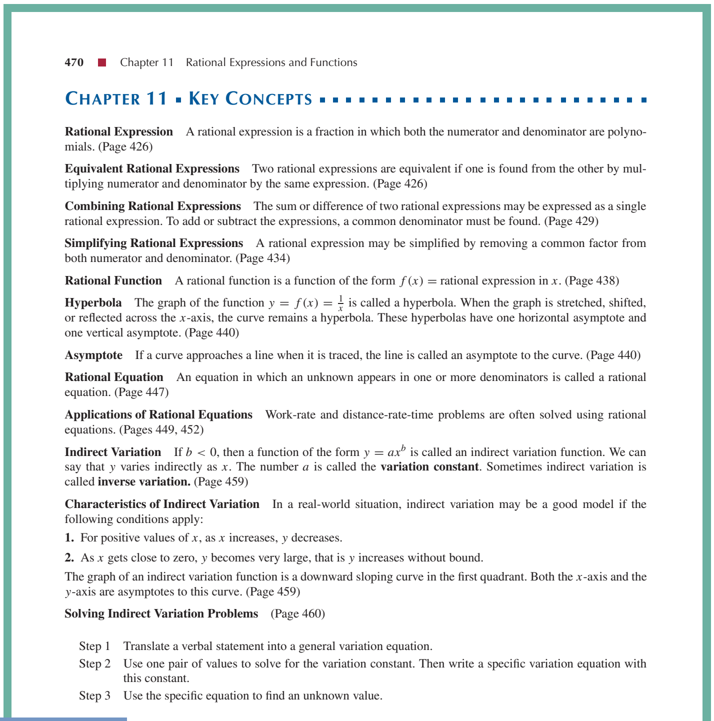
3.

**In Exercises 12–15, express the domain and range of each function on a number line and in set builder notation.**

# End of Chapter

## Key Concepts

A chapter summary is provided at the end of every chapter. A list of all essential information, with definitions and page references, serves as a study and review tool for students before moving on to the Review Exercises and Chapter Test.



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### CHAPTER 11 ▪ KEY CONCEPTS

**Rational Expression** A rational expression is a fraction in which both the numerator and denominator are polynomials. (Page 426)

**Equivalent Rational Expressions** Two rational expressions are equivalent if one is found from the other by multiplying numerator and denominator by the same expression. (Page 426)

**Combining Rational Expressions** The sum or difference of two rational expressions may be expressed as a single rational expression. To add or subtract the expressions, a common denominator must be found. (Page 429)

**Simplifying Rational Expressions** A rational expression may be simplified by removing a common factor from both numerator and denominator. (Page 434)

**Rational Function** A rational function is a function of the form $f(x) =$ rational expression in $x$. (Page 438)

**Hyperbola** The graph of the function $y = f(x) = \frac{1}{x}$ is called a hyperbola. When the graph is stretched, shifted, or reflected across the $x$-axis, the curve remains a hyperbola. These hyperbolas have one horizontal asymptote and one vertical asymptote. (Page 440)

**Asymptote** If a curve approaches a line when it is traced, the line is called an asymptote to the curve. (Page 440)

**Rational Equation** An equation in which an unknown appears in one or more denominators is called a rational equation. (Page 447)

**Applications of Rational Equations** Work-rate and distance-rate-time problems are often solved using rational equations. (Pages 449, 452)

**Indirect Variation** If $b < 0$, then a function of the form $y = ax^b$ is called an indirect variation function. We can say that $y$ varies indirectly as $x$. The number $a$ is called the **variation constant**. Sometimes indirect variation is called **inverse variation.** (Page 459)

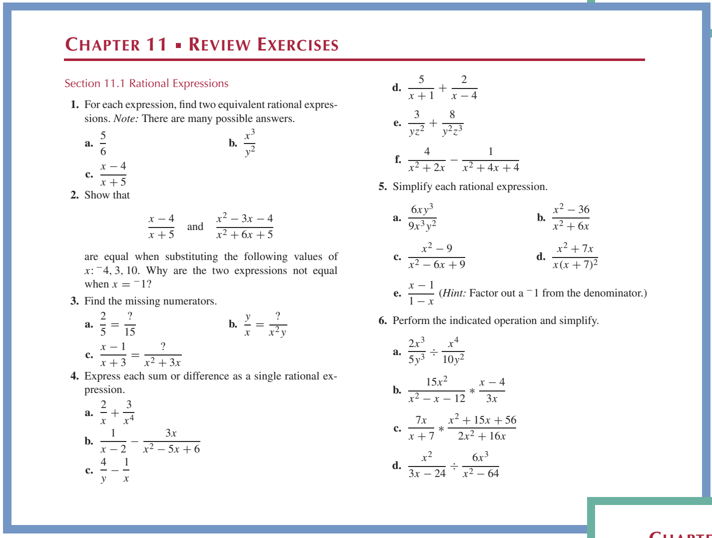
**Characteristics of Indirect Variation** In a real-world situation, indirect variation may be a good model if the following conditions apply:

1. For positive values of $x$, as $x$ increases, $y$ decreases.
2. As $x$ gets close to zero, $y$ becomes very large, that is $y$ increases without bound.

The graph of an indirect variation function is a downward sloping curve in the first quadrant. Both the $x$-axis and the $y$-axis are asymptotes to this curve. (Page 459)

**Solving Indirect Variation Problems** (Page 460)

Step 1 Translate a verbal statement into a general variation equation.
Step 2 Use one pair of values to solve for the variation constant. Then write a specific variation equation with this constant.
Step 3 Use the specific equation to find an unknown value.



### CHAPTER 11 ▪ REVIEW EXERCISES

Section 11.1 Rational Expressions

**1.** For each expression, find two equivalent rational expressions. *Note:* There are many possible answers.
  **a.** $\frac{5}{6}$ **b.** $\frac{x^3}{y^2}$
  **c.** $\frac{x-4}{x+5}$

**2.** Show that

$$\frac{x-4}{x+5} \quad \text{and} \quad \frac{x^2-3x-4}{x^2+6x+5}$$

are equal when substituting the following values of $x$: $^-4, 3, 10$. Why are the two expressions not equal when $x = {}^-1$?

**3.** Find the missing numerators.
  **a.** $\frac{2}{5} = \frac{?}{15}$ **b.** $\frac{y}{x} = \frac{?}{x^2y}$
  **c.** $\frac{x-1}{x+3} = \frac{?}{x^2+3x}$

**4.** Express each sum or difference as a single rational expression.
  **a.** $\frac{2}{x} + \frac{3}{x^4}$
  **b.** $\frac{1}{x-2} - \frac{3x}{x^2-5x+6}$
  **c.** $\frac{4}{y} - \frac{1}{x}$
  **d.** $\frac{5}{x+1} + \frac{2}{x-4}$
  **e.** $\frac{3}{yz^2} + \frac{8}{y^2z^3}$
  **f.** $\frac{4}{x^2+2x} - \frac{1}{x^2+4x+4}$

**5.** Simplify each rational expression.
  **a.** $\frac{6xy^3}{9x^3y^2}$ **b.** $\frac{x^2-36}{x^2+6x}$
  **c.** $\frac{x^2-9}{x^2-6x+9}$ **d.** $\frac{x^2+7x}{x(x+7)^2}$
  **e.** $\frac{x-1}{1-x}$ (*Hint:* Factor out a $^-1$ from the denominator.)

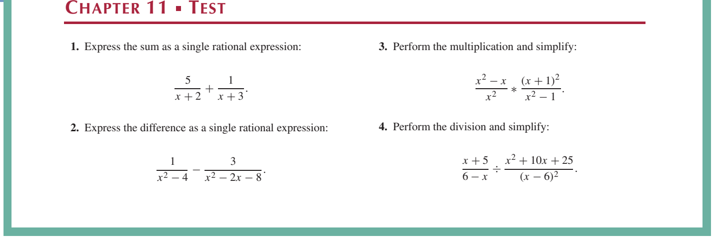
**6.** Perform the indicated operation and simplify.
  **a.** $\frac{2x^3}{5y^3} \div \frac{x^4}{10y^2}$
  **b.** $\frac{15x^2}{x^2-x-12} * \frac{x-4}{3x}$
  **c.** $\frac{7x}{x+7} * \frac{x^2+15x+56}{2x^2+16x}$
  **d.** $\frac{x^2}{3x-24} \div \frac{6x^3}{x^2-64}$

## Review Exercises

Review Exercises appear at the end of each chapter. These provide a complete review of all topics covered and allow students to link together sections and learning objectives.

## Chapter Test

The Chapter Test exercises are designed to simulate a possible test of the material in the chapter.



### CHAPTER 11 ▪ TEST

**1.** Express the sum as a single rational expression:

$$\frac{5}{x+2} + \frac{1}{x+3}.$$

**2.** Express the difference as a single rational expression:

$$\frac{1}{x^2-4} - \frac{3}{x^2-2x-8}.$$

**3.** Perform the multiplication and simplify:

$$\frac{x^2-x}{x^2} * \frac{(x+1)^2}{x^2-1}.$$

**4.** Perform the division and simplify:

$$\frac{x+5}{6-x} \div \frac{x^2+10x+25}{(x-6)^2}.$$