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New Challenges • New Solutions

Integration of Technology

by Joanne S. Lockwood

Many of the students in my algebra classes have previously studied the material in this course but they did not succeed at learning it via the traditional approach. Because I believed that they would respond more favorably to a different approach, I started incorporating the graphing calculator into the course. I am delighted with the results. The advantages I discovered of teaching a graphing-calculator-dependent algebra class were incorporated into the structure and features of the textbook *Exploring Introductory and Intermediate Algebra: A Graphing Approach.*

Throughout this text we have used a graphing calculator to help students make connections between abstract mathematical concepts and a concrete representation provided by technology. This blending of concept and technology encourages students to think about and use multiple representations of a concept. For instance, for appropriate examples within the text, we have provided both an algebraic solution and a graphical representation of the solution. This provides the student with a visual, concrete representation of the algebraic solution. In other examples, an algebraic verification of a graphing calculator solution is presented, as for Example 5 on page 637, shown below.

EXAMPLE 5

According to the *Keenan Report #1,* "Exchanges in the Internet Economy," the actual and projected consumer-to-consumer transactions, in billions, from online auctions, such as eBay, are as shown in the chart below. An equation that models these data is $y = 0.6144x^2 - 7.7658x + 23.9343$, where y is the consumer-to-consumer transactions, in billions of dollars, in year x, and $x = 10$ represents the year 2000. During which year does the model predict that transactions will reach $30 billion?

Year	1995	1996	1997	1998	1999	2000	2001
Transactions (in billions)	$0	$.03	$.13	$.75	$3.5	$7.6	$13.1

As this example illustrates, the inclusion of technology facilitates computation in order to focus on analysis rather than manipulation. This enables stu-dents to examine concepts that otherwise would be too difficult or time consuming if the technol-ogy were not available.

Shown below is another example related to quadratic equations.

EXAMPLE 7

The fuel efficiency of an automobile engine varies with the speed at which the car is driven. Because the fuel efficiency depends on the speed, fuel efficiency is a function of the speed of the car. The results of an automotive study comparing the speed x, in miles per hour, and the average fuel efficiency y, in miles per gallon, are shown in the table below.

x	15	20	25	30	35	40	45	50	55	60	65	70
y	22.1	25.3	27.3	28.2	28.6	28.8	29.7	30.0	30.2	28.6	27.2	25.3

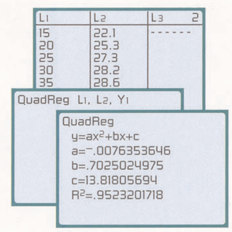
a. Use regression to determine a quadratic model for these data.
b. Use the equation to predict the fuel efficiency when the car is driven at 48 miles per hour.
c. Use the equation to predict the speeds at which the fuel efficiency of a car is 27 miles per gallon.
d. Find the vertex of the graph of the equation. Round coordinates to the nearest tenth. Explain the meaning of the vertex in the context of the data.

Solution

a. Use a graphing calculator to perform the regression.

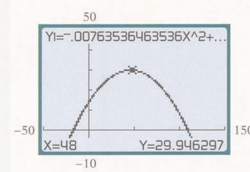
The quadratic regression equation is given in the form $y = ax^2 + bx + c$.

To the nearest millionth, the equation is $y = -0.007635x^2 + 0.702502x + 13.818057$.



b. Graph the regression equation. Use the TRACE feature to find the value of y when $x = 48$.

The coordinates printed at the bottom of the screen are approximately (48, 29.95).



The equation predicts that the fuel efficiency when the car is driven at 48 miles per hour is approximately 29.95 miles per gallon.

Note that students are asked to predict a y value for a given x value and to predict an x value for a given y value. Answering these questions results in greater understanding of the domain and range of a function, which they are better able to visualize with the equation graphed on a calculator. They must also find the vertex of the graph of the model equation and explain the meaning of the vertex in the context of the data. Using the calculator to determine the vertex results in less time spent algebraically calculating the maximum and more time analyzing what that point represents in the context of the problem.

As the examples above illustrate, we have integrated numerous data analysis exercises in the textbook and encourage students to use technology to assist them in deriving meaningful conclusions about the data. The very large numbers inherent in most real data is not a deterrent when a graphing calculator is used. Some of the topics of the exercises in the sections cited above are traffic patterns, online shopping, internet radio stations, agriculture, banking, business, social security, labor unions, and sports. Students are more motivated and interested when presented with problems involving meaningful data.

The use of technology also enables instructors to use the discovery approach from time to time. The example below is from the *Exploration* on page 644.

EXPLORATION

1. *The Effect of the Discriminant on Solutions of Quadratic Equations and x-Intercepts of Parabolas*

As you work through this project, record your results in a table such as the one shown here.

$y = ax^2 + bx + c$	Number of x-intercepts of the graph	Solutions of the equation $ax^2 + bx + c = 0$	Nature of the solutions of $ax^2 + bx + c = 0$	Value of $b^2 - 4ac$
$y = 4x^2 - 4x + 1$				
$y = 4x^2 - 4x + 3$				
$y = 4x^2 - 4x - 2$				
$y = -x^2 + 6x - 9$				
$y = -x^2 + 6x - 10$				
$y = -x^2 + 6x - 5$				

a. For each quadratic equation in two variables given in the first column, use a graphing calculator to determine the number of x-intercepts of the graph of the equation (column 2).

b. Calculate the solutions of the corresponding quadratic equation in one variable (column 3).

c. Record the nature of the solutions to the quadratic equation (column 4). Is it a double root? Two real number solutions? Two complex number solutions?

d. The quantity $b^2 - 4ac$ is called the discriminant. It is the quantity under the radical sign in the quadratic formula. Record in the last column the value of the determinant of each quadratic equation.

e. Make a conjecture as to the effect of the determinant on the solutions of a quadratic equation and the number of x-intercepts of a parabola. Test your conjecture on other quadratic equations and determine whether your conjecture is correct for these equations. If not, modify your conjecture and try a few more equations.

All of the references above are from Chapter 10, *Quadratic Functions and Quadratic Inequalities*. These same features are included in the other chapters of *Exploring Introductory and Intermediate Algebra: A Graphing Approach*. Here are some references from Chapter 11, *Exponential and Logarithmic Functions*.

Solve a problem algebraically and check it using a graph ing calculator, Examples 7 and 8 on pages 720 and 721

Solve a problem using a graphing calculator and check it algebraically, Examples 3 and 5 on pages 770 and 773

Model equations and use the equation to analyze data, Example 3, page 732

Data analysis exercises, pages 738-739

The discovery approach, Explorations, Exponential Functions of the Form f(x) = ab-x and Negative Values of a in Exponential Functions of the Form f(x) = abx, page 728

See also the *Student Activity Manual* which accompanies the text. It offers many graphing calculator activities to supplement classroom presentations, for example, Section 11.3, Part B.

Joanne S. Lockwood is the co-author of Exploring Intermediate Algebra: A Graphing Approach.