



Motivating Liberal Arts Math Students Through Problem Solving

by Joanne S. Lockwood

The First Week of Class

The students in my Mathematics for the Liberal Arts classes have had little success in their previous math courses. Therefore, my goal during the first week of class is to present them with a different experience and to engage them in the learning process. This means discarding the traditional lecture format and presenting them with ideas and concepts they have not encountered in previous courses.

Focus on Problem Solving, not Solutions

One way to explore new ideas and concepts is to focus on problem solving. It is a skill the students will use throughout the course, in other courses, and in their lives. It is a topic that the majority of students have not read about or analyzed, but one in which they can become involved. It also lends itself to discussion and verbal descriptions. My students generally are inarticulate in mathematics. By presenting them with problems at the beginning of the semester that are easy for them to discuss, students become better prepared to talk about more advanced topics later in the course. In the problem-solving arena, I can ask fairly simple questions such as “What strategy did you use to solve this problem?” or “Did anyone take a different approach to solving this problem?” I let them know that the correct answer to a problem is less important than their descriptions of the processes they went through to arrive at their solutions. At this stage, the focus is on *solving*, not on the *solution*. This comes as quite a surprise to most students and alleviates some of the anxiety they always feel when the “right answer” is the all-important grade earner.

Cooperative Learning

A unit on problem solving also lends itself to cooperative learning. I ask small groups of students to work together. Three or four students per group works nicely for most activities. With larger groups, some students may not participate fully; with fewer than three students, and there may not be a sufficient diversity of ideas presented. Students do learn from each other. My students have experienced a great deal of frustration in mathematics; having other students to work with helps them to keep working on a problem whereas, if left on their own, they might have given up. Another advantage is that working in groups reinforces verbalizing concepts and ideas.

Activity-Based Instruction

One challenge for me was coming up with activities related to problem solving that were at the appropriate level of difficulty, would involve the students, and would reinforce the skills and concepts I wanted them to learn. Many of the successful activities are included in the text *Mathematical Excursions*. This textbook incorporates a number of features that can be used in the hands-on-learning approach. For example, a feature entitled “Excursions” precedes every exercise set. The first Excursion in the chapter on problem solving introduces the game of sprouts. I have my students work in small groups on this activity. It is definitely engaging and activity oriented, but it is also an excellent experience in problem solving and in applying their problem-solving skills.



Excursion

The Game of Sprouts by John H. Conway

The mathematician John H. Conway has created several games that are easy to play but complex enough to provide plenty of mental stimulation. In 1967, Conway, along with Michael Paterson, created the two-person, paper-and-pencil game of *Sprouts*. After more than 30 years, the game of *Sprouts* has not been completely analyzed. Here are the rules for *Sprouts*.


Rules for Sprouts

1. A few spots (dots) are drawn on a piece of paper.
2. Players alternate turns. A turn consists of drawing a curve between two spots or drawing a curve that starts at a spot and ends at the same spot. The active player then places a new spot on the new curve. No curve may pass through a previously drawn spot. No curve may cross itself or a previously drawn curve.
3. A spot with no rays emanating from it has three lives. A spot with one ray emanating from it has two lives. A spot with two rays emanating from it has one life. A spot is dead and cannot be used when it has three rays emanating from it. See the figure at the left.
4. The winner is the last player who is able to draw a curve.

If *Sprouts* is played with just one spot (called *1-Spot Sprouts*), then the second player always wins. See the following analysis of *1-Spot Sprouts*.

(continued)


A spot with 3 lives.


A spot with 2 lives.


A spot with 1 life.


A dead spot.

The status of a spot in the game of *Sprouts*.

Another feature that was written with these same goals in mind is entitled “Extensions” and appears at the end of each exercise set. For example, in the first section of the problem-solving chapter, under the heading “Critical Thinking,” is an exercise that asks students to use inductive reasoning to predict the next letter in the list O, T, T, F, F, S, S, E, These, too, are excellent exercises for students to work on in small groups. It can be beneficial to then have students share their ideas and strategies with the class as a whole.

Features for Student Involvement

If you are interested in engaging your students in activities and discussions during the initial class meetings of your Math for the Liberal Arts course, check out these features in *Mathematical Excursions*.

Section 1.1, Excursion, *The Game of Sprouts by John H. Conway*, pages 10-11

Section 1.1, Extensions, Critical Thinking Exercises 45-47, which involve inductive reasoning, pages 13-14

Section 1.2, Excursion, *Polygonal Numbers*, pages 22-23

Section 1.2, Extensions, Cooperative Learning Exercises 27-30, which involve Bodes Rule and inductive reasoning, pages 26-27

Section 1.2, Extensions, Explorations Exercise 31, which discusses the Tower of Hanoi, page 27

Section 1.3, Excursion, *Routes on a Probability Demonstrator*, page 37

Section 1.3, Extensions, Cooperative Learning Exercises 51-52, which require use of problem-solving strategies, pages 41-42

Joanne S. Lockwood is the co-author of *Mathematical Excursions*.