Beginning Algebra with Arithmetic Review

The Maricopa Mathematics Consortium

The authors' work is guided by six student outcomes. These are referred to as **CREATE** outcomes.

CREATE

CONNECT Students employ a variety of methods (visual, symbolic, numeric, verbal, and more) to represent and explore mathematical ideas. They construct and apply models that connect mathematics to the world, and evaluate the soundness of their methodologies and results.

REASON Students demonstrate clear reasoning in analyzing information. They develop and apply logical thinking skills to formulate and support conclusions.

EXPRESS Students read, write, listen to, and speak mathematics both individually and in teams.

APPRECIATE Students value the power of mathematics. They are confident, flexible, and persistent lifelong learners and users of mathematics.

TAP INTO TECHNOLOGY Students discover the benefits and limitations of current technologies. They utilize technologies as resources for learning and problem solving.

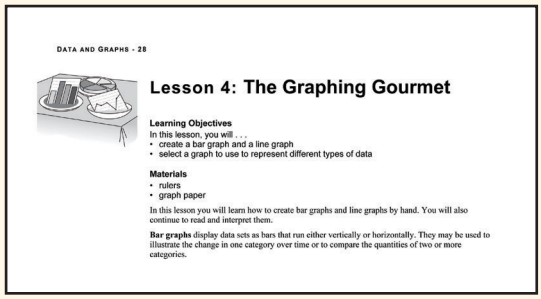
ESTABLISH A FOUNDATION Foundation skills provide a basis for continual learning. Students acquire and develop a core of content-specific knowledge and abilities, as well as strategies for learning.

Pedagogical Blueprint

Each module is comprised of several Lessons and uses the following effective features to help students achieve the **CREATE** *outcomes.*

Introduction ▶

Each lesson introduces the idea/concept to be studied. It lists the objectives and the materials needed.



DATA AND GRAPHS - 28

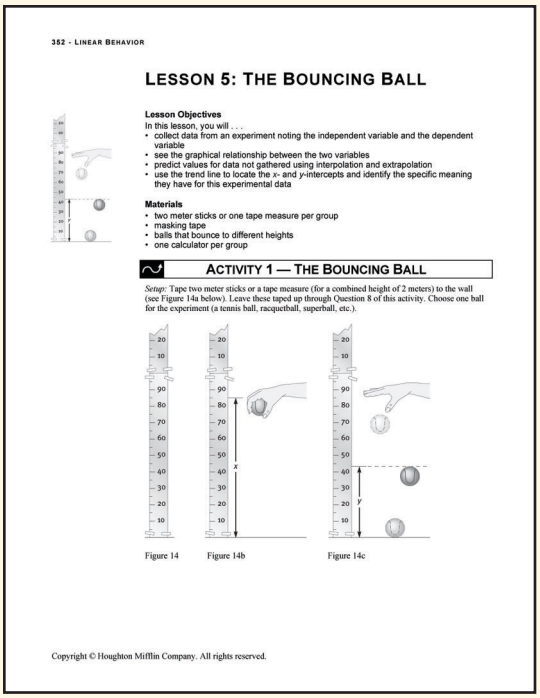
Lesson 4: The Graphing Gourmet

Learning Objectives
In this lesson, you will . . .
- create a bar graph and a line graph
- select a graph to use to represent different types of data

Materials
- rulers
- graph paper

In this lesson you will learn how to create bar graphs and line graphs by hand. You will also continue to read and interpret them.

Bar graphs display data sets as bars that run either vertically or horizontally. They may be used to illustrate the change in one category over time or to compare the quantities of two or more categories.



352 - LINEAR BEHAVIOR

LESSON 5: THE BOUNCING BALL

Lesson Objectives
In this lesson, you will . . .
- collect data from an experiment noting the independent variable and the dependent variable
- see the graphical relationship between the two variables
- predict values for data not gathered using interpolation and extrapolation
- use the trend line to locate the *x*- and *y*-intercepts and identify the specific meaning they have for this experimental data

Materials
- two meter sticks or one tape measure per group
- masking tape
- balls that bounce to different heights
- one calculator per group

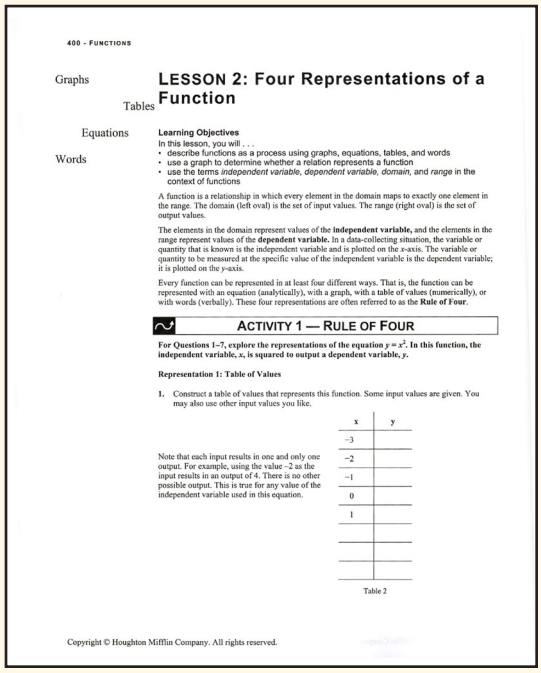
ACTIVITY 1 — THE BOUNCING BALL

Setup: Tape two meter sticks or a tape measure (for a combined height of 2 meters) to the wall (see Figure 14a below). Leave these taped up through Question 8 of this activity. Choose one ball for the experiment (a tennis ball, racquetball, superball, etc.).

Figure 14 Figure 14b Figure 14c

◀ Activities

Each lesson is organized around student activities that are introduced by short expositions. Data is presented (or student-gathered), modules are given (or student-created), and questions are asked which are specifically designed to lead students to a real-world, mathematical connection to the concept. Because the activities are part of the course material, it is easy to develop a daily classroom routine of active student involvement.



400 - FUNCTIONS

Graphs
Tables
Equations
Words

LESSON 2: Four Representations of a Function

Learning Objectives
In this lesson, you will . . .
- describe functions as a process using graphs, equations, tables, and words
- use a graph to determine whether a relation represents a function
- use the terms *independent variable, dependent variable, domain,* and *range* in the context of functions

A function is a relationship in which every element in the domain maps to exactly one element in the range. The domain (left oval) is the set of input values. The range (right oval) is the set of output values.

The elements in the domain represent values of the **independent variable,** and the elements in the range represent values of the **dependent variable.** In a data-collecting situation, the variable or quantity that is known is the independent variable and is plotted on the *x*-axis. The variable or quantity to be measured at the specific value of the independent variable is the dependent variable; it is plotted on the *y*-axis.

Every function can be represented in at least four different ways. That is, the function can be represented with an equation (analytically), with a graph, with a table of values (numerically), or with words (verbally). These four representations are often referred to as the **Rule of Four**.

ACTIVITY 1 — RULE OF FOUR

For Questions 1–7, explore the representations of the equation $y = x^2$. In this function, the independent variable, *x*, is squared to output a dependent variable, *y*.

Representation 1: Table of Values

1. Construct a table of values that represents this function. Some input values are given. You may also use other input values you like.

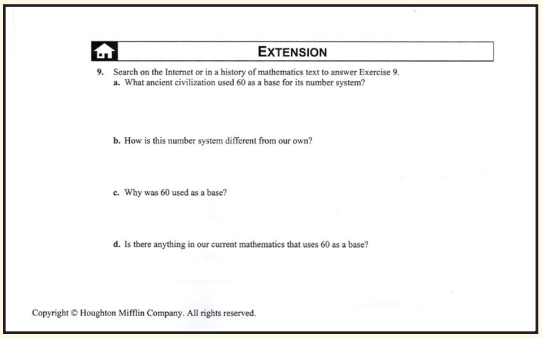
Note that each input results in one and only one output. For example, using the value –2 as the input results in an output of 4. There is no other possible output. This is true for any value of the independent variable used in this equation.

x	y
–3	
–2	
–1	
0	
1	

Table 2

Rule of Four ▶

Students use four ways to express and explore mathematical ideas: graphically, numerically, symbolically, and verbally. In this way, students make a multitude of connections around specific mathematical concepts.



EXTENSION

9. Search on the Internet or in a history of mathematics text to answer Exercise 9.
   a. What ancient civilization used 60 as a base for its number system?

   b. How is this number system different from our own?

   c. Why was 60 used as a base?

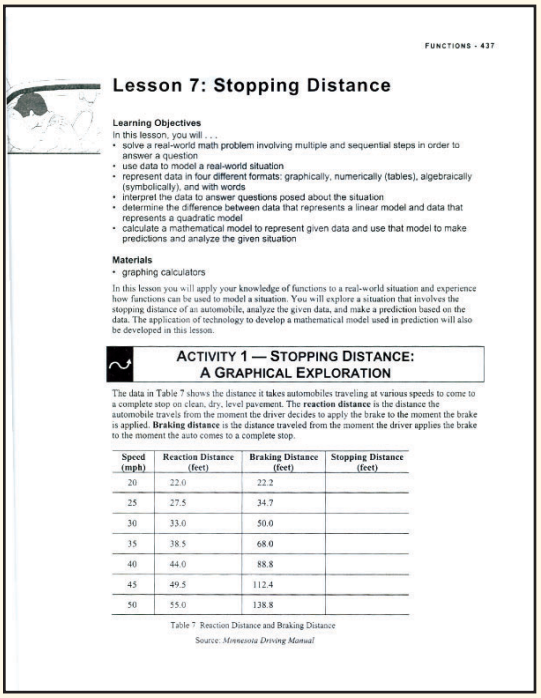
   d. Is there anything in our current mathematics that uses 60 as a base?

◀ Writing

Writing is an integrated feature. Students are asked to write explanations in their own words and express the real-world meaning of something written mathematically. Extension boxes build on concepts previously covered by asking students to work through more challenging problems.

Technology

Students use graphing calculators daily throughout all the modules. Any graphing utility can be used, but it should have data lists, data plotting, and regression capability. The features of the TI-83 graphing calculator fit the material nicely.



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Lesson 7: Stopping Distance

Learning Objectives

In this lesson, you will . . .

- solve a real-world math problem involving multiple and sequential steps in order to answer a question
- use data to model a real-world situation
- represent data in four different formats: graphically, numerically (tables), algebraically (symbolically), and with words
- interpret the data to answer questions posed about the situation
- determine the difference between data that represents a linear model and data that represents a quadratic model
- calculate a mathematical model to represent given data and use that model to make predictions and analyze the given situation

Materials

- graphing calculators

In this lesson you will apply your knowledge of functions to a real-world situation and experience how functions can be used to model a situation. You will explore a situation that involves the stopping distance of an automobile, analyze the given data, and make a prediction based on the data. The application of technology to develop a mathematical model used in prediction will also be developed in this lesson.

ACTIVITY 1 — STOPPING DISTANCE: A GRAPHICAL EXPLORATION

The data in Table 7 shows the distance it takes automobiles traveling at various speeds to come to a complete stop on clean, dry, level pavement. The **reaction distance** is the distance the automobile travels from the moment the driver decides to apply the brake to the moment the brake is applied. **Braking distance** is the distance traveled from the moment the driver applies the brake to the moment the auto comes to a complete stop.

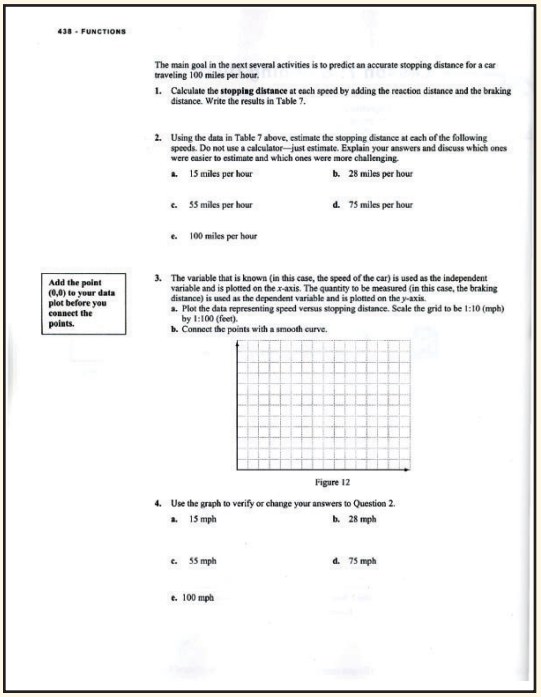
Speed (mph)	Reaction Distance (feet)	Braking Distance (feet)	Stopping Distance (feet)
20	22.0	22.2	
25	27.5	34.7	
30	33.0	50.0	
35	38.5	68.0	
40	44.0	88.8	
45	49.5	112.4	
50	55.0	138.8	

Table 7 Reaction Distance and Braking Distance

Source: *Minnesota Driving Manual*

◀ ▼Graphing Interpretation Problems

These problems are integrated throughout and are closely related to writing as students see and explain how a real-world model and mathematical model relate.



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The main goal in the next several activities is to predict an accurate stopping distance for a car traveling 100 miles per hour.

1. Calculate the **stopping distance** at each speed by adding the reaction distance and the braking distance. Write the results in Table 7.

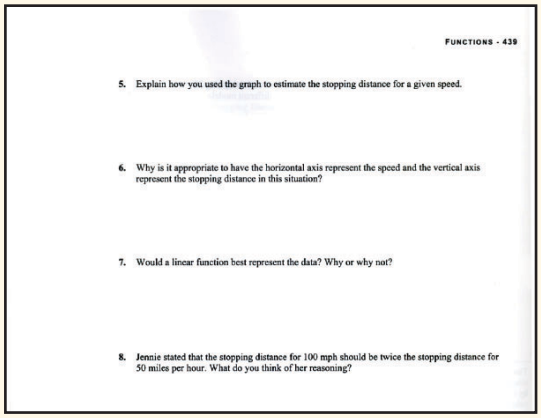
2. Using the data in Table 7 above, estimate the stopping distance at each of the following speeds. Do not use a calculator—just estimate. Explain your answers and discuss which ones were easier to estimate and which ones were more challenging.
   a. 15 miles per hour
   b. 28 miles per hour
   c. 55 miles per hour
   d. 75 miles per hour
   e. 100 miles per hour

Add the point (0,0) to your data plot before you connect the points.

3. The variable that is known (in this case, the speed of the car) is used as the independent variable and is plotted on the *x*-axis. The quantity to be measured (in this case, the braking distance) is used as the dependent variable and is plotted on the *y*-axis.
   a. Plot the data representing speed versus stopping distance. Scale the grid to be 1:10 (mph) by 1:100 (feet).
   b. Connect the points with a smooth curve.

Figure 12

4. Use the graph to verify or change your answers to Question 2.
   a. 15 mph
   b. 28 mph
   c. 55 mph
   d. 75 mph
   e. 100 mph



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5. Explain how you used the graph to estimate the stopping distance for a given speed.

6. Why is it appropriate to have the horizontal axis represent the speed and the vertical axis represent the stopping distance in this situation?

7. Would a linear function best represent the data? Why or why not?

8. Jennie stated that the stopping distance for 100 mph should be twice the stopping distance for 50 miles per hour. What do you think of her reasoning?

Homework ▶ ▼

The homework problems are designed to reinforce and extend the concepts of the lesson. They provide opportunities for problem solving. In the homework, students make progress toward accomplishment of the **CREATE** outcomes, as well as skill development. Each problem has a unique contribution to make to a student's gain in skills and understanding.

HOMEWORK

1. Imagine that you received a report of transactions, but *one digit* in each number was smudged (■) and unreadable. For each of the following, determine whether the result will be positive, zero, negative, or impossible to determine.

a. −57■7 − 43■ **b.** 6■9 − (−3■7)

c. ■18 − (−■30) **d.** −1■32 − (−79■)

2. Write the subtraction of integers problem that is represented by these chips.

3. Draw the chips which represent the problem and solution to 5 − (−4).

4. Use *The Return of the Sweater* scenario to explain the problem −25 − (−50).

5. The subtraction problem 5 − 8 and the addition problem 5 + (−8) both give the result −3. What different meaning does each problem represent?

20. Use subtraction to model each of the following.

a. The high temperature in Phoenix one summer day was 108°F. The low temperature that same day was 85°F. What was the difference between the high and the low temperatures that day?

b. The high temperature in Fairbanks one winter day was − 25°F. The low temperature that same day was −33°F. What was the difference between the high and the low temperatures that day?

21. On Monday morning you purchased stock. By the close of the day, the price rose $2. On Tuesday it rose $6. Wednesday, it fell $11. Thursday it fell another $17 and on Friday it rose $7. Represent the situation with addition and subtraction of integers. Determine the net change in the price of the stock for the week.

22. One year Russell hiked to the top of Mt. Whitney (elev. 4417 meters) in California. Later that year he visited Badwater, CA (elev. −86 meters) in Death Valley. What was the total difference in elevation that Russell experienced that year?

23. Clarissa was on a weight-loss program one year. She weighed 165 pounds to start. The first month Clarissa lost 6 pounds. The second month she lost 7 pounds. The third month she went on vacation and gained 1 pound. Represent the situation with addition and subtraction of integers. How much did Clarissa weigh at the end of the third month?

In Exercises 6–15, perform the indicated operation without a calculator.

6. − 5 − 8 **7.** − 5 − (−7)

8. 10 − (−12) **9.** −28 − 12

10. 32 − (−18) **11.** −18 − 42

12. − 227 − 15 **13.** −106 − (−10)

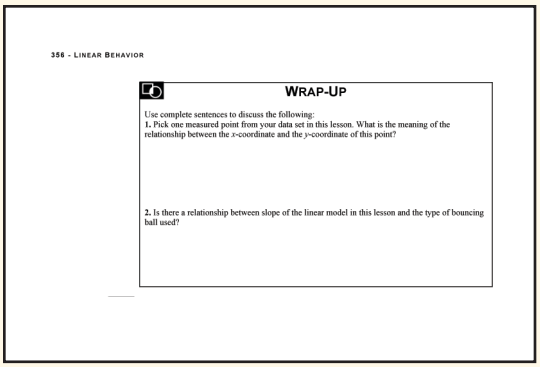
14. −15 − (−5) + 7 **15.** −18 + 42 − (−3) − 6

16. Write a rule for subtracting a negative integer from any integer.

17. Write a rule for subtracting a positive integer from any integer.

18. Write three different subtraction problems where the difference is −1.

19. Write three different subtraction problems, involving integers with opposite signs, where the difference is 6.



◀ Wrap-Up

Often prompted by questions, this feature asks students to write their own summary, encouraging them to be active participants in the learning process.

Assessment

Most students appreciate the opportunity to demonstrate what they know in a variety of ways. For this reason we encourage a variety of assessments. In each lesson, students are encouraged to self-assess by completing the activities, the Wrap-Up boxes, and the homework. Assessment ideas for instructors include quizzes, projects, research activities, and writing assignments, as well as tests.