Chapter Opening Features

Chapter Opener

Each chapter begins with a **Chapter Opener** that illustrates a specific application of a concept from the chapter. This application references an exercise in the chapter where students solve a problem related to the chapter opener topic.

57. ***Wedding Expenses*** The function $C(t) = 17t^2 + 128t + 5900$ models the average cost of a wedding reception and the function $W(t) = 38t^2 + 291t + 15{,}208$ models the average cost of a wedding, where $t = 0$ represents the year 1990 and $0 \le t \le 12$. The rational function

$$R(t) = \frac{C(t)}{W(t)} = \frac{17t^2 + 128t + 5900}{38t^2 + 291t + 15{,}208}$$

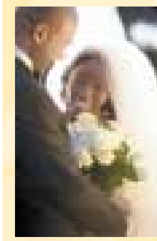
gives the relative cost of the reception compared to the cost of a wedding.

a. Use $R(t)$ to estimate the relative cost of the reception compared to the cost of a wedding for the years $t = 0$, $t = 7$, and $t = 12$. Round your results to the nearest tenth of a percent.

page 373

Chapter 4

Polynomial and Rational Functions



Wedding Expenses

In this chapter you will study polynomial and rational functions. These types of functions have many practical applications. They can be used to model and analyze wedding expenses, as shown below.

The average cost of a wedding was $15,208 in 1990, $19,104 in 1997, and about $23,000 in 2001. The table to the right lists some of the average costs associated with a wedding in 1990 and in 1997.

The polynomial function $D(t) = 4.8t + 793$ models the average cost of a wedding dress, and the function $W(t) = 38t^2 + 291t + 15{,}208$ models the average cost of a wedding, where $t = 0$ represents the year 1990 and $0 \le t \le 12$. The rational function

$$R(t) = \frac{4.8t + 793}{38t^2 + 291t + 15{,}208}$$

represents the relative cost of a wedding dress compared to the cost of a wedding. For $t = 0$, $t = 7$, and $t = 12$, we find that $R(0) \approx 5.2\%$, $R(7) \approx 4.3\%$, and $R(12) \approx 3.5\%$.

Thus, although the average price of a wedding dress has steadily increased over the last few years, the relative cost of a wedding dress, compared to the average cost of a wedding, has decreased.

Another wedding expense application is given in Exercise 57, page 373.

Category	Average cost, 1990	Average cost, 1997
Flowers	$478	$756
Music	$882	$830
Rehearsal dinner	$501	$698
Wedding dress	$794	$823
Reception	$5900	$7635

Source: Bride's Magazine, 1997 Millennium Report (**EmeraldWeddings.com**)

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Chapter Prep Quiz *Do these exercises to prepare for Chapter 1.*

1. Simplify: $\sqrt{12}$ [P.6]*
2. Simplify: $3(2x - 1) - 4(3x - 2)$ [P.3]
3. Factor: $x^2 - 16x + 64$ [P.4]
4. Factor: $x^4 + 2x^2 - 3$ [P.4]
5. Express "the distance between a real number x and 5 is less than 3" using absolute value notation. [P.1]
6. Simplify: $\dfrac{-4 + \sqrt{-16}}{2}$ [P.7]
7. Write $\dfrac{x}{x+1} - 2$ as a rational expression. [P.5]
8. Evaluate $\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ with $a = 2$, $b = -4$, and $c = 1$. [P.6]
9. Find $\{x \mid x \ge -1\} \cup \{x \mid x < 5\}$. [P.1]
10. Find $\{x \mid x \ge 3\} \cap \{x \mid x > 1\}$. [P.1]

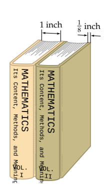
Problem Solving Strategies

Verifying Results

One important aspect of problem solving involves the process of checking to see if your results satisfy the conditions of the original problem. This process will be especially important in this chapter when you solve an equation or an inequality.

Here is an example that illustrates the importance of checking your results. The problem seems easy, but many students fail to get the correct answer on their first attempt.

Two volumes of the series *Mathematics: Its Content, Methods, and Meaning* are on a shelf, with no space between the volumes. Each volume is 1 inch thick without its covers. Each cover is $\frac{1}{8}$ inch thick. A bookworm bores horizontally from the first page of Volume I to the last page of Volume II. How far does the bookworm travel?



Once you have obtained your solution, try to check it by closely examining two books placed as shown above. Check to make sure you have the proper starting and ending positions. The correct answer is $\frac{1}{4}$ inch.

*This is a reference to the section that corresponds to this problem. For example, [P.6] stands for Chapter P, Section 6.

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Prep Quiz and Problem Solving Strategies

Chapter Prep Quizzes occur at the beginning of each chapter and test students on previously covered concepts that are required in order to succeed in the upcoming chapter. Next to each question, in brackets, is a reference to the section of the text that contains the concepts related to the question to allow students to refer back for help. All answers are provided in Answers to Selected Exercises.

Problem Solving Strategies give students insight into successful problem-solving strategies and help students better understand how they are used.

Aufmann Interactive Method (AIM)

This text is written in a style that encourages the student to interact with the textbook.

An Interactive Approach

Applied College Algebra uses an interactive approach that provides students with an opportunity to try a skill as it is presented. This feature can be used by instructors as an easy way to immediately check for student understanding and to actively engage students in practicing concepts as they are presented.

For each numbered Example within a section, there is a similar *Check Your Progress* problem for the student to work. Each *Check Your Progress* problem references a page in the back of the text where the full solution is presented—rather than just the answer. By including the full solution, the *Check Your Progress* exercises provide students immediate feedback on their understanding of the concepts and serve as additional examples for students to refer to while studying.

Question/Answer

At various places during a discussion, we ask students to respond to a **Question** about the material being presented. This question encourages students to pause and think about the mathematics. To make sure students do not miss important information, and to help those students studying independently, the **Answer** to the question is provided as a footnote at the bottom of the page.

414 CHAPTER 5 Exponential and Logarithmic Functions

TAKE NOTE

Pay close attention to these properties. Note that

$$\log_b(MN) \neq \log_b M \cdot \log_b N$$

and

$$\log_b \frac{M}{N} \neq \frac{\log_b M}{\log_b N}$$

Also,

$$\log_b(M + N) \neq \log_b M + \log_b N$$

In fact, the expression $\log_b(M + N)$ cannot be expanded at all.

Properties of Logarithms

In the following properties, b, M, and N are positive real numbers ($b \neq 1$).

Product property	$\log_b(MN) = \log_b M + \log_b N$
Quotient property	$\log_b \frac{M}{N} = \log_b M - \log_b N$
Power property	$\log_b(M^p) = p \log_b M$
Logarithm-of-each-side property	$M = N$ implies $\log_b M = \log_b N$
One-to-one property	$\log_b M = \log_b N$ implies $M = N$

QUESTION Is it true that $\ln 5 + \ln 10 = \ln 50$?

The above properties of logarithms are often used to rewrite logarithmic expressions in an equivalent form.

EXAMPLE 1 Rewrite Logarithmic Expressions

Use the properties of logarithms to express the following logarithms in terms of logarithms of x, y, and z.

a. $\log_5(xy^2)$ **b.** $\log_b \frac{2\sqrt{y}}{z^5}$

Solution

a. $\log_5(xy^2) = \log_5 x + \log_5 y^2$ ▪ Product property
$= \log_5 x + 2 \log_5 y$ ▪ Power property

b. $\log_b \frac{2\sqrt{y}}{z^5} = \log_b(2\sqrt{y}) - \log_b z^5$ ▪ Quotient property
$= \log_b 2 + \log_b \sqrt{y} - \log_b z^5$ ▪ Product property
$= \log_b 2 + \log_b y^{1/2} - \log_b z^5$ ▪ Replace $\sqrt{y}$ with $y^{1/2}$
$= \log_b 2 + \frac{1}{2}\log_b y - 5 \log_b z$ ▪ Power property

CHECK YOUR PROGRESS 1 Use the properties of logarithms to express $\ln \frac{z^3}{\sqrt{xy}}$ in terms of logarithms of x, y, and z.

Solution *See page S25.*

The properties of logarithms are also used to rewrite expressions that involve several logarithms as a single logarithm.

ANSWER Yes. By the product property, $\ln 5 + \ln 10 = \ln(5 \cdot 10)$.

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Section 5.3

Check Your Progress 1, *page 414*

$$\ln \frac{z^3}{\sqrt{xy}} = \ln z^3 - \ln\sqrt{xy}$$
$$= \ln z^3 - \ln(xy)^{1/2}$$
$$= 3 \ln z - \frac{1}{2}\ln(xy)$$
$$= 3 \ln z - \frac{1}{2}(\ln x + \ln y)$$
$$= 3 \ln z - \frac{1}{2}\ln x - \frac{1}{2}\ln y$$

page S25

AIM for Success

Welcome to *Applied College Algebra*. As you begin this course, we know two important facts: (1) We want you to succeed. (2) You want to succeed. To do that requires an effort from each of us. For the next few pages, we are going to show you what is required of you to achieve that success and how you can use the features of this text to be successful.

Motivation

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways mathematics can benefit you. But, in the end, the motivation must come from you. On the first day of class, it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

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AIM for Success Student Preface

This "how to use this book" student preface explains what is required of a student to be successful in mathematics and how this text has been designed to foster student success through the Aufmann Interactive Method (AIM). *AIM for Success* can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills. There are suggestions for teaching this lesson in the *Instructor's Resource Manual* and on the *Class Prep CD*.

Real Data and Applications

Applications

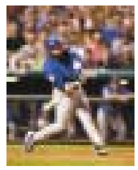
One way to motivate an interest in mathematics is through applications. This carefully integrated, applied approach generates student awareness of the value of algebra as a relevant real-life tool.

Applications in this text are taken from many disciplines to address the diverse interests and backgrounds of students. Topics include agriculture, business, carpentry, chemistry, construction, Earth science, health science, education, manufacturing, nutrition, real estate, sports, and sociology.

Wherever appropriate, applications use problem-solving strategies to solve practical problems.

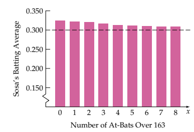
Applications of Quadratic and Rational Inequalities

Quadratic inequalities and rational inequalities are often used to solve applied problems. Here are a few examples.



EXAMPLE 5 Solve an Application Involving Batting Averages

Near the end of May 2002, Sammy Sosa had 53 hits out of 163 at-bats. At that time his batting average was approximately 0.325. If Sosa goes into a batting slump in which he gets no hits, how many more at-bats will it take for his batting average to fall below 0.300?



Solution A baseball player's batting average is determined by dividing the player's number of hits by the number of times the player has been at bat. Let x be the number of additional at-bats that Sosa takes over 163. During this period his batting average will be $\frac{53}{163 + x}$, and we wish to solve

$$\frac{53}{163 + x} < 0.300$$

This rational inequality can be solved by using a sign diagram or the critical value method, but there is an easier method. In this application we know that $163 + x$ is positive. Thus if we multiply each side of the above inequality by $163 + x$, we will obtain the linear inequality $53 < 48.9 + 0.300x$, with the condition that x is a positive integer. Solving this inequality produces

$$53 < 48.9 + 0.300x$$
$$4.1 < 0.300x$$
$$x > 13.\overline{6}$$

Because x must be a positive integer, Sosa's average will fall below 0.300 if he goes hitless for 14 or more at-bats.

CHECK YOUR PROGRESS 5 Assume Sammy Sosa has 53 hits out of 163 at-bats. If Sosa goes into a hitting streak in which he gets a hit every time he bats, how many more at-bats will it take for his batting average to exceed 0.350?

Solution *See page S9.*

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Life and Health Sciences

16. ***Generation of Garbage*** According to the U.S. Environmental Protection Agency, the amount of garbage generated per person has been increasing over the last few decades. The following table shows the per capita garbage, in pounds per day, generated in the United States.

Year, t	1960	1970	1980	1990	2000
Pounds per day, p	2.66	3.27	3.61	4.00	4.30

Represent the year 1960 by $t = 60$.

a. Use a graphing utility to find a linear model and a logarithmic model for the data. Use t as the independent variable (domain) and p as the dependent variable (range).

b. Examine the correlation coefficients of the two regression models to determine which model provides a better fit for the data.

c. Use the model you selected in part **b.** to predict the amount of garbage that will be generated per capita per day in 2005. Round to the nearest hundredth of a pound.

17. ***The Henderson-Hasselbach Function*** The scientists Henderson and Hasselbach determined that the pH of blood is a function of the ratio q of the amounts of bicarbonate and carbonic acid in the blood.

a. Use a graphing utility and the data in the following table to determine a linear model and a logarithmic model for the data. Use q as the independent variable (domain) and pH as the dependent variable (range). State the correlation coefficient for each model. Round a and b to 5 decimal places and r to 6 decimal places. Which model provides the better fit for the data?

q	7.9	12.6	31.6	50.1	79.4
pH	7.0	7.2	7.6	7.8	8.0

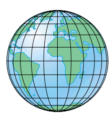
b. Use the model you chose in part **a.** to find the q-value associated with a pH of 8.2. Round to the nearest tenth.

18. ***World Population*** The following table lists the years in which the world's population first reached 3, 4, 5, and 6 billion.

World Population Milestones

1960	3 billion
1974	4 billion
1987	5 billion
1999	6 billion

Source: Time Almanac 2002, page 708.



a. Find an exponential model for the data in the table. Let $x = 0$ represent the year 1960.

b. Use the model to predict the year in which the world's population will first reach 7 billion.

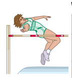
19. ***Panda Population*** One estimate gives the world panda population as 3200 in 1980 and 590 in 2000.

a. Find an exponential model for the data and use the model to predict the year in which the panda population p will be reduced to 200. (Let $t = 0$ represent the year 1980.)

b. Because the exponential model in part **a.** fits the data perfectly, does this mean that the model will accurately predict future panda populations? Explain.

Sports and Recreation

20. ***Olympic Records*** The following table shows the Olympic gold medal distances for the women's high jump from 1968 to 2000.



Women's Olympic High Jump, 1968 to 2000

Year	Distance	Year	Distance
1968	5 ft $11\frac{3}{4}$ in.	1984	6 ft $7\frac{1}{2}$ in.
1972	6 ft $3\frac{5}{8}$ in.	1988	6 ft 8 in.
1976	6 ft 4 in.	1992	6 ft $7\frac{1}{2}$ in.
1980	6 ft $5\frac{1}{2}$ in.	1996	6 ft $8\frac{3}{4}$ in.
		2000	6 ft 7 in.

Source: Time Almanac 2002

Represent the year 1968 by 68.

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Real Data

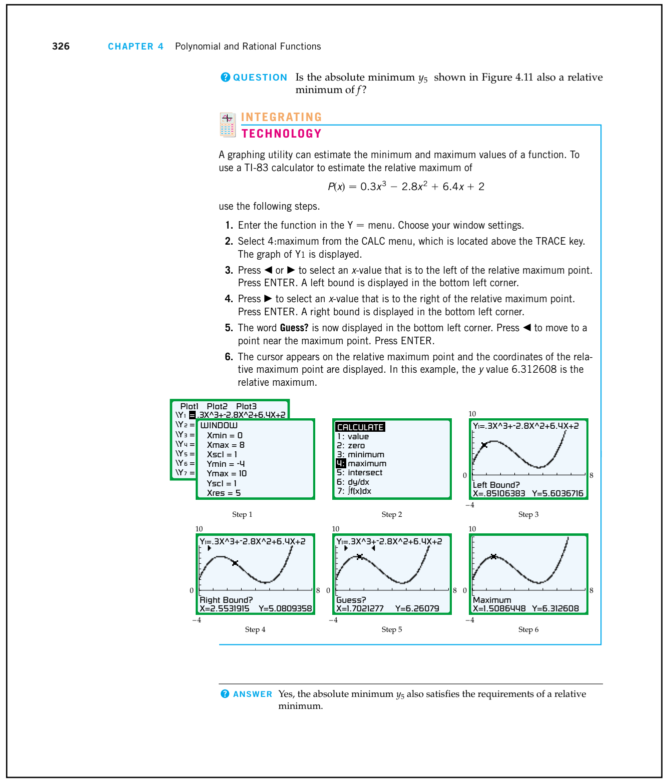
Real data examples and exercises, identified by [pie chart icon], ask students to analyze and solve problems taken from actual situations. Students often work with tables, graphs, and charts drawn from a variety of disciplines.

Technology

Integrating Technology

The Integrating Technology feature contains discussions that can be used to further explore a concept using technology. Some introduce technology as an alternative way to solve certain problems and others provide suggestions for using a calculator to solve certain problems and applications.

Additionally, optional graphing calculator examples and exercises (identified by ) are presented throughout the text.



326 CHAPTER 4 Polynomial and Rational Functions

QUESTION Is the absolute minimum y_5 shown in Figure 4.11 also a relative minimum of f?

INTEGRATING TECHNOLOGY

A graphing utility can estimate the minimum and maximum values of a function. To use a TI-83 calculator to estimate the relative maximum of

$$P(x) = 0.3x^3 - 2.8x^2 + 6.4x + 2$$

use the following steps.

1. Enter the function in the Y = menu. Choose your window settings.
2. Select 4:maximum from the CALC menu, which is located above the TRACE key. The graph of Y1 is displayed.
3. Press ◀ or ▶ to select an x-value that is to the left of the relative maximum point. Press ENTER. A left bound is displayed in the bottom left corner.
4. Press ▶ to select an x-value that is to the right of the relative maximum point. Press ENTER. A right bound is displayed in the bottom left corner.
5. The word **Guess?** is now displayed in the bottom left corner. Press ◀ to move to a point near the maximum point. Press ENTER.
6. The cursor appears on the relative maximum point and the coordinates of the relative maximum point are displayed. In this example, the y value 6.312608 is the relative maximum.

Step 1 Step 2 Step 3

Step 4 Step 5 Step 6

ANSWER Yes, the absolute minimum y_5 also satisfies the requirements of a relative minimum.

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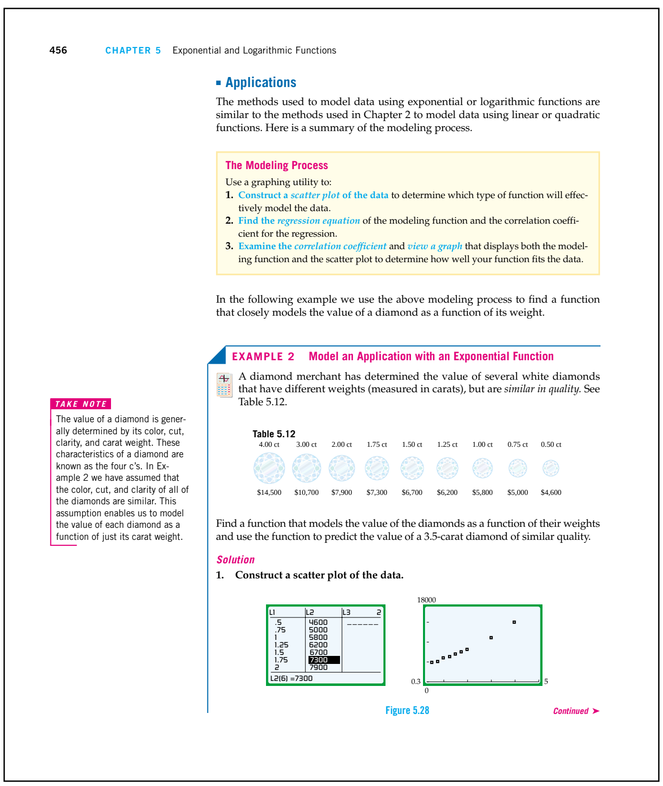

456 CHAPTER 5 Exponential and Logarithmic Functions

Applications

The methods used to model data using exponential or logarithmic functions are similar to the methods used in Chapter 2 to model data using linear or quadratic functions. Here is a summary of the modeling process.

The Modeling Process

Use a graphing utility to:

1. Construct a *scatter plot* of the data to determine which type of function will effectively model the data.
2. Find the *regression equation* of the modeling function and the correlation coefficient for the regression.
3. Examine the *correlation coefficient* and *view a graph* that displays both the modeling function and the scatter plot to determine how well your function fits the data.

In the following example we use the above modeling process to find a function that closely models the value of a diamond as a function of its weight.

TAKE NOTE

The value of a diamond is generally determined by its color, cut, clarity, and carat weight. These characteristics of a diamond are known as the four c's. In Example 2 we have assumed that the color, cut, and clarity of all of the diamonds are similar. This assumption enables us to model the value of each diamond as a function of just its carat weight.

EXAMPLE 2 Model an Application with an Exponential Function

A diamond merchant has determined the value of several white diamonds that have different weights (measured in carats), but are *similar in quality*. See Table 5.12.

Table 5.12

4.00 ct	3.00 ct	2.00 ct	1.75 ct	1.50 ct	1.25 ct	1.00 ct	0.75 ct	0.50 ct
$14,500	$10,700	$7,900	$7,300	$6,700	$6,200	$5,800	$5,000	$4,600

Find a function that models the value of the diamonds as a function of their weights and use the function to predict the value of a 3.5-carat diamond of similar quality.

Solution

1. **Construct a scatter plot of the data.**

Figure 5.28

Continued ➤

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Modeling

Special modeling sections, which rely heavily on the use of a graphing calculator, are incorporated within the text. These optional sections introduce the idea of a mathematical model using various real-world data sets, which further motivate students and help them see the relevance of mathematics.

Student Pedagogy

This text was designed to be an understandable resource for students. Special emphasis was given to readability and effective pedagogical use of color to highlight important words and concepts.

Icons

The 

icons at each objective head remind students of the many and varied additional resources available for each objective.

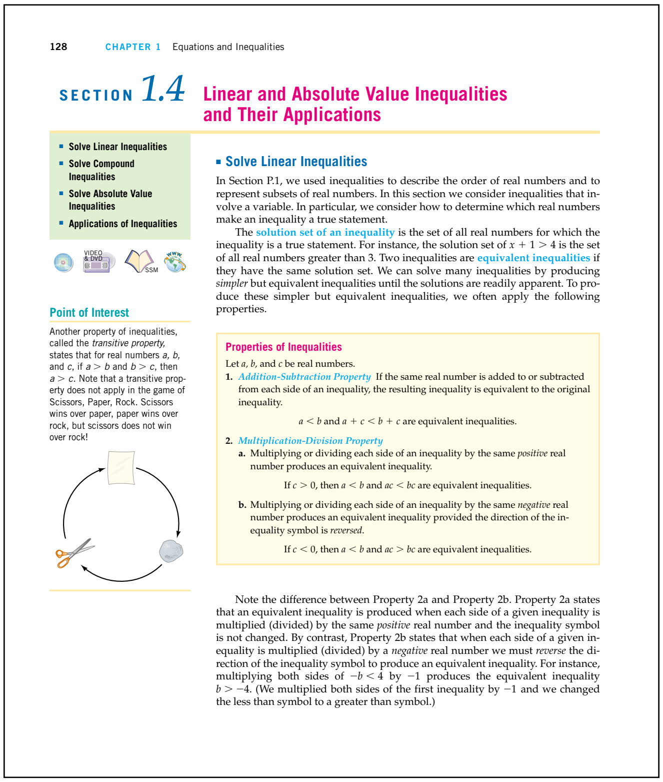
Key Terms and Important Concepts

A blue bold font is used whenever a **key term** is first introduced.

Important Concepts are presented in yellow boxes in order to highlight these concepts and serve as an easy-to-find reference.

Point of Interest

These margin notes contain interesting comments about mathematics, its history, or its application.



128 CHAPTER 1 Equations and Inequalities

SECTION 1.4 Linear and Absolute Value Inequalities and Their Applications

- Solve Linear Inequalities
- Solve Compound Inequalities
- Solve Absolute Value Inequalities
- Applications of Inequalities

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Point of Interest

Another property of inequalities, called the *transitive property*, states that for real numbers a, b, and c, if $a > b$ and $b > c$, then $a > c$. Note that a transitive property does not apply in the game of Scissors, Paper, Rock. Scissors wins over paper, paper wins over rock, but scissors does not win over rock!

■ Solve Linear Inequalities

In Section P.1, we used inequalities to describe the order of real numbers and to represent subsets of real numbers. In this section we consider inequalities that involve a variable. In particular, we consider how to determine which real numbers make an inequality a true statement.

The **solution set of an inequality** is the set of all real numbers for which the inequality is a true statement. For instance, the solution set of $x + 1 > 4$ is the set of all real numbers greater than 3. Two inequalities are **equivalent inequalities** if they have the same solution set. We can solve many inequalities by producing *simpler* but equivalent inequalities until the solutions are readily apparent. To produce these simpler but equivalent inequalities, we often apply the following properties.

Properties of Inequalities

Let a, b, and c be real numbers.

1. *Addition-Subtraction Property* If the same real number is added to or subtracted from each side of an inequality, the resulting inequality is equivalent to the original inequality.

   $a < b$ and $a + c < b + c$ are equivalent inequalities.

2. *Multiplication-Division Property*
   a. Multiplying or dividing each side of an inequality by the same *positive* real number produces an equivalent inequality.

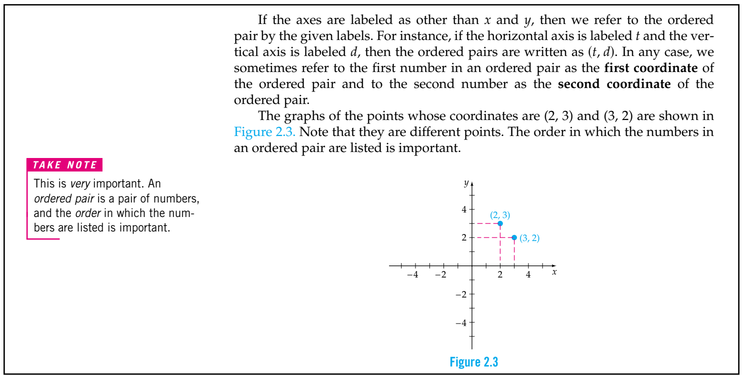
      If $c > 0$, then $a < b$ and $ac < bc$ are equivalent inequalities.

   b. Multiplying or dividing each side of an inequality by the same *negative* real number produces an equivalent inequality provided the direction of the inequality symbol is *reversed*.

      If $c < 0$, then $a < b$ and $ac > bc$ are equivalent inequalities.

Note the difference between Property 2a and Property 2b. Property 2a states that an equivalent inequality is produced when each side of a given inequality is multiplied (divided) by the same *positive* real number and the inequality symbol is not changed. By contrast, Property 2b states that when each side of a given inequality is multiplied (divided) by a *negative* real number we must *reverse* the direction of the inequality symbol to produce an equivalent inequality. For instance, multiplying both sides of $-b < 4$ by -1 produces the equivalent inequality $b > -4$. (We multiplied both sides of the first inequality by -1 and we changed the less than symbol to a greater than symbol.)

page 128



If the axes are labeled as other than x and y, then we refer to the ordered pair by the given labels. For instance, if the horizontal axis is labeled t and the vertical axis is labeled d, then the ordered pairs are written as (t, d). In any case, we sometimes refer to the first number in an ordered pair as the **first coordinate** of the ordered pair and to the second number as the **second coordinate** of the ordered pair.

The graphs of the points whose coordinates are (2, 3) and (3, 2) are shown in Figure 2.3. Note that they are different points. The order in which the numbers in an ordered pair are listed is important.

TAKE NOTE

This is *very* important. An *ordered pair* is a pair of numbers, and the *order* in which the numbers are listed is important.

Figure 2.3

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Take Note

These margin notes alert students to a point requiring special attention or are used to highlight the concept under discussion.

Exercises

Topics for Discussion

Topics for Discussion provide questions related to key concepts of the section. Instructors can use these to initiate class discussions or to ask students to write about concepts presented in the section.

Exercises

The exercise sets of *Applied College Algebra* emphasize skill building, skill maintenance, and applications. Concept-based writing or developmental exercises have also been integrated within the exercise sets.

Icons identify appropriate writing, group, data analysis, and graphing calculator exercises.

Applications

Whenever possible, applications of mathematics are emphasized. Application exercises are grouped under one of five categories:

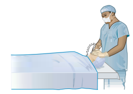
Business and Economics *Life and Health Sciences*
Social Sciences *Sports and Recreation*
Physical Sciences and Engineering

Each application exercise has a title that further describes the particular application.

5.2 Logarithmic Functions and Their Applications 411

Life and Health Sciences

68. *Medicine* In anesthesiology it is necessary to accurately estimate the body surface area of a patient. One formula for estimating body surface area (*BSA*) was developed by Edith Boyd (University of Minnesota Press, 1935). Her formula for the *BSA* (in square meters) of a patient of height *H* (in centimeters) and weight *W* (in grams) is



$$BSA = 0.0003207 \cdot H^{0.3} \cdot W^{(0.7285 - 0.0188 \log W)}$$

Use Boyd's formula to estimate the body surface area of a patient with the given weight and height. Round to the nearest hundredth of a square meter.

a. $W = 110$ pounds (49,895.2 grams), $H = 5$ feet 4 inches (162.56 centimeters)

b. $W = 180$ pounds (81,646.6 grams), $H = 6$ feet 1 inch (185.42 centimeters)

c. $W = 40$ pounds (18,143.7 grams), $H = 29$ inches (73.66 centimeters)

Sports and Recreation

69. *World Records in the Discus Throw* The function $d(t) = -41.71 + 25.76 \ln t$ approximates the world record distance, in meters, for the men's discus throw for the years 1965 to 1985. The year 1965 is represented by $t = 65$.

a. According to the function $d(t)$, what was the world record distance in 1975 and in 1984? Round to the nearest hundredth meter.

b. Use the Internet to check the world record distance in the men's discus throw for the current year. How does this actual result compare with the distance predicted by the function $d(t)$?

Physical Sciences and Engineering

70. *Astronomy* Astronomers measure the apparent brightness of a star by a unit called the **apparent magnitude.** This unit was created in the second century B.C. when the Greek astronomer Hipparchus classified the relative brightness of several stars. In his list he assigned the number 1 to the stars that appeared to be the brightest (Sirius, Vega, and Deneb). They are first-magnitude stars. Hipparchus assigned the number 2 to all the stars in the Big Dipper. They are second-magnitude stars. The following table shows the relationship between a star's brightness relative to a first-magnitude star and the star's apparent magnitude. Notice from the table that a first-magnitude star appears in the sky to be about 2.51 times as bright as a second-magnitude star.

Brightness relative to a first-magnitude star x	Apparent magnitude $M(x)$
1	1
$\frac{1}{2.51}$	2
$\frac{1}{6.31} \approx \frac{1}{2.51^2}$	3
$\frac{1}{15.85} \approx \frac{1}{2.51^3}$	4
$\frac{1}{39.82} \approx \frac{1}{2.51^4}$	5
$\frac{1}{100} \approx \frac{1}{2.51^5}$	6

The following logarithmic function gives the apparent magnitude $M(x)$ of a star as a function of its brightness x.

$$M(x) = -2.51 \log x + 1, \quad 0 < x \leq 1$$

Topics for Discussion

1. If $m > n$, must $\log_b m > \log_b n$?
2. For what values of x is $\ln x > \log x$?
3. What is the domain of $f(x) = \log(x^2 + 1)$? Explain why the graph of f does not have a vertical asymptote.
4. The subtraction $3 - 5$ does not have an answer if we require that the answer be positive. Keep this idea in mind as you work the rest of this exercise.

Press the MODE key of a TI-83 graphing calculator and choose "Real" from the menu. Now use the calculator to evaluate $\log(-2)$. What output is given by the calculator? Press the MODE key and choose "$a + bi$" from the menu. Now use the calculator to evaluate $\log(-2)$. What output is given by the calculator? Write a sentence or two that explain why the output is different for these two evaluations.

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Prepare for Section 5.4

66. Use the definition of a logarithm to write the exponential equation $3^6 = 729$ in logarithmic form. [5.2]

67. Use the definition of a logarithm to write the logarithmic equation $\log_5 625 = 4$ in exponential form. [5.2]

68. Use the definition of a logarithm to write the exponential equation $a^{x+2} = b$ in logarithmic form. [5.2]

69. Solve for x: $4a = 7bx + 2cx$ [1.1]

70. Solve for x: $165 = \frac{300}{1 + 12x}$ [1.3]

71. Solve for x: $A = \frac{100 + x}{100 - x}$ [1.3]

Explorations

1. *Logarithmic Scales* Sometimes **logarithmic scales** are used to better view a collection of data that span a wide range of values. For instance, consider the table below, which lists the approximate masses of various marine creatures in grams. Next we have attempted to plot the masses on a number line.

Animal	Mass (g)
Rotifer	0.000000006
Dwarf goby	0.30
Lobster	15,900
Leatherback turtle	851,000
Giant squid	1,820,000
Whale shark	4,700,000
Blue whale	120,000,000

masses of the different animals?

c. If the data points for two animals on the logarithmic number line are 1 unit apart, how do the animals' masses compare? What if the points are 2 units apart?

2. *Logarithmic Scales* The distances of the planets in our solar system from the sun are given in the table below.

Planet	Distance (million km)
Mercury	58
Venus	108
Earth	150
Mars	228
Jupiter	778
Saturn	1427
Uranus	2871
Neptune	4497
Pluto	5913

a. Draw a number line with an appropriate scale to plot the distances.

b. Draw a second number line, this time plotting the logarithm (base 10) of each distance.

c. Which number line do you find more helpful to compare the different distances?

d. If two distances are 3 units apart on the logarithmic number line, how do the distances of the corresponding planets compare?

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Exercises to Prepare for the Next Section

Every section's exercise set (except for the last section of a chapter) contain exercises that allow students to practice the previously learned skills and concepts students will need to be successful in the next section. Next to each question, in brackets, is a reference to the section of the text that contains the concepts related to the question for students to easily review. All answers are provided in Answers to Selected Exercises.

Explorations are provided at the end of each exercise set and are designed to encourage students to do research and write about what they have learned. These Explorations generally emphasize critical thinking skills and can be used as collaborative learning exercises or as extra credit assignments.

End of Chapter

Chapter Summary

At the end of each chapter there is a Chapter Summary that includes **Key Terms** and **Essential Concepts and Formulas** that were covered in the chapter. These chapter summaries provide a single point of reference as the student prepares for a test. Each key term and concept references the page number from the lesson where the term or concept was first introduced.

Chapter True/False Exercises

Following each chapter summary are true/false exercises. These exercises are intended to help students understand concepts and can be used to initiate class discussions.

Chapter Review Exercises

Review exercises are found at the end of each chapter. These exercises are selected to help the student integrate all of the topics presented in the chapter.

Chapter 5 True/False Exercises

In Exercises 1 to 11, answer true or false. If the statement is false, explain why it is false or give an example to show the statement is false.

1. If $7^x = 40$, then $\log_7 40 = x$.

2. If $\log_4 x = 3.1$, then $4^{3.1} = x$.

7. A population that is growing exponentially at a continuous rate of 10% per year will double its population within 5 years.

8. If a radioactive material has a half-life of 15 days, all of the material will decay in 30 days.

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Chapter 5 Review Exercises

In Exercises 1 to 12, solve each equation. Do not use a calculator.

1. $\log_5 25 = x$ **2.** $\log_3 81 = x$

3. $\ln e^3 = x$ **4.** $\ln e^{\pi} = x$

17. $f(x) = 2^x - 3$ **18.** $f(x) = 2^{(x-3)}$

19. $f(x) = \frac{1}{3}\log x$ **20.** $f(x) = 3\log x^{1/3}$

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Chapter 5 Test

1. Evaluate *without* using a calculator: $\log_3 \frac{1}{27}$

2. Use the change-of-base formula and a calculator to approximate $\log_4 12$. Round your result to the nearest ten thousandth.

7. Write $e^{t/4} = a$ in logarithmic form.

8. Write $\log_b \frac{z^2}{y^3\sqrt{x}}$ in terms of logarithms of x, y, and z.

9. Solve $5^x = 22$. Round to the nearest ten thousandth.

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Cumulative Review Exercises

1. Solve $|x - 4| \le 2$. Write the solution set using interval notation.

2. Solve $\frac{x}{2x - 6} \ge 1$. Write the solution set using set-builder notation.

4. The height, in feet, of a ball released with an initial upward velocity of 44 feet per second and at an initial height of 8 feet is given by $h(t) = -16t^2 + 44t + 8$, where t is the time in seconds after the ball is released. Find the maximum height the ball will reach.

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CHAPTER 5 Summary 477

Chapter 5 Summary

Key Terms

acid [p. 421]
acidity of a solution [p. 420]
alkaline solution [p. 421]
apparent magnitude [p. 411]
base [p. 421]
Benford's Law [p. 412]
biological diversity [p. 452]
carbon dating [p. 446]
carrying capacity [p. 468]
catenary [p. 399]
common logarithm [p. 407]
concave upward (downward) [p. 454]
continuous growth rate [p. 440]
decibel level [p. 424]
e (base of natural exponential function) [p. 390]
exponential decay [p. 440]
exponential equation [p. 427]
exponential form [p. 401]
exponential function [p. 385]
exponential growth [p. 440]
factorial function [p. 397]
growth rate constant [p. 468]
half-life [p. 444]
inflection point [p. 472]
logarithm [p. 401]
logarithmic equation [p. 431]
logarithmic form [p. 401]
logarithmic function [p. 401]
logarithmic scale [pp. 417/426]
logistic growth model [p. 466]
Malthusian growth model [p. 466]
natural exponential function [p. 391]
natural logarithm [p. 407]
Newton's Law of Cooling [p. 446]
nomogram [p. 425]
pH of a solution [p. 421]
power function [p. 464]
p-wave [p. 419]
Richter scale magnitude of an earthquake [p. 417]
s-wave [p. 420]
zero-level earthquake [p. 417]

Essential Concepts and Formulas

- **Exponential Functions**
  - For $b > 0$ and $b \neq 1$, the exponential function $f(x) = b^x$ has the following properties:
    f has the set of real numbers as its domain and the set of positive real numbers as its range.
    f has a graph with a y-intercept of $(0, 1)$, and the graph passes through $(1, b)$.
    f is a one-to-one function, and the graph of f is a smooth continuous curve that is asymptotic to the x-axis.
    f is an increasing function if $b > 1$.
    f is a decreasing function if $0 < b < 1$. **[p. 387]**
  - As n increases without bound, $\left(1 + \frac{1}{n}\right)^n$ approaches an irrational number denoted by e. The value of e accurate to eight decimal places is 2.71828183. **[p. 390]**
  - The function defined by $f(x) = e^x$ is called the natural exponential function. **[p. 391]**
- **Logarithmic Functions**
  - *Definition of a Logarithm* If $x > 0$ and b is a positive constant ($b \neq 1$), then $y = \log_b x$ if and only if $b^y = x$. **[p. 401]**
  - The *exponential form* of $y = \log_b x$ is $b^y = x$. **[p. 401]**
  - The *logarithmic form* of $b^y = x$ is $y = \log_b x$. **[p. 401]**
  - *Basic Logarithmic Properties* **[p. 402]**
    $\log_b b = 1$ $\log_b 1 = 0$ $\log_b b^x = x$ $b^{\log_b x} = x$
  - For all positive real numbers b, $b \neq 1$, the logarithmic function defined by $f(x) = \log_b x$ has the following properties:
    f has the set of positive real numbers as its domain and the set of real numbers as its range.
    f has a graph with an x-intercept of $(1, 0)$. The graph passes through $(b, 1)$.

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Chapter Test

The Chapter Test exercises are designed to simulate a possible test of the material in the chapter.

Cumulative Review Exercises

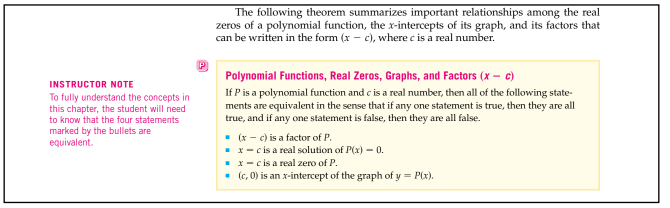
Cumulative Review Exercises, which appear at the end of each chapter (beginning with Chapter 1), allow students to refresh previously developed skills and concepts.

The answers to all Chapter Review Exercises, all Chapter Test exercises, and all Cumulative Review Exercises are given in Answers to Selected Exercises. Along with the answer, there is a reference to the section that pertains to each exercise. This further illustrates how the text supports students while they are studying and preparing for exams.

Instructor's Annotated Edition

The Instructor's Annotated Edition includes the following features:

Instructor Notes give suggestions for teaching concepts, warnings about common student errors, or historical notes.



The following theorem summarizes important relationships among the real zeros of a polynomial function, the x-intercepts of its graph, and its factors that can be written in the form $(x - c)$, where c is a real number.

INSTRUCTOR NOTE
To fully understand the concepts in this chapter, the student will need to know that the four statements marked by the bullets are equivalent.

Polynomial Functions, Real Zeros, Graphs, and Factors $(x - c)$

If P is a polynomial function and c is a real number, then all of the following statements are equivalent in the sense that if any one statement is true, then they are all true, and if any one statement is false, then they are all false.

- $(x - c)$ is a factor of P.
- $x = c$ is a real solution of $P(x) = 0$.
- $x = c$ is a real zero of P.
- $(c, 0)$ is an x-intercept of the graph of $y = P(x)$.

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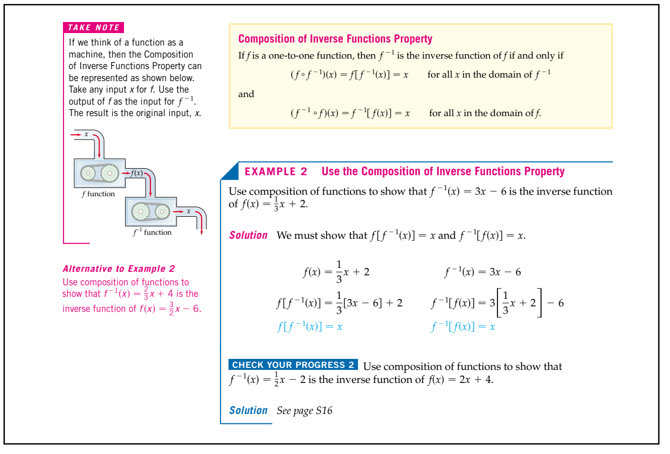
Next to many of the graphs or tables in the text, there is a P icon that indicates that a Microsoft **PowerPoint® slide** of that figure is available. These slides (along with PowerPoint Viewer) are available on the *Class Prep CD* and can also be downloaded from our web site at **math.college.hmco.com/instructors**. These slides can also be printed as transparency masters.

An **Alternative to Example** note accompanies every example and offers an additional example for an instructor to use in class.



TAKE NOTE
If we think of a function as a machine, then the Composition of Inverse Functions Property can be represented as shown below. Take any input x for f. Use the output of f as the input for f^{-1}. The result is the original input, x.

Alternative to Example 2
Use composition of functions to show that $f^{-1}(x) = \frac{2}{3}x + 4$ is the inverse function of $f(x) = \frac{3}{2}x - 6$.

Composition of Inverse Functions Property

If f is a one-to-one function, then f^{-1} is the inverse function of f if and only if

$$(f \circ f^{-1})(x) = f[f^{-1}(x)] = x \qquad \text{for all } x \text{ in the domain of } f^{-1}$$

and

$$(f^{-1} \circ f)(x) = f^{-1}[f(x)] = x \qquad \text{for all } x \text{ in the domain of } f.$$

EXAMPLE 2 Use the Composition of Inverse Functions Property

Use composition of functions to show that $f^{-1}(x) = 3x - 6$ is the inverse function of $f(x) = \frac{1}{3}x + 2$.

Solution We must show that $f[f^{-1}(x)] = x$ and $f^{-1}[f(x)] = x$.

$$f(x) = \frac{1}{3}x + 2 \qquad f^{-1}(x) = 3x - 6$$

$$f[f^{-1}(x)] = \frac{1}{3}[3x - 6] + 2 \qquad f^{-1}[f(x)] = 3\left[\frac{1}{3}x + 2\right] - 6$$

$$f[f^{-1}(x)] = x \qquad f^{-1}[f(x)] = x$$

CHECK YOUR PROGRESS 2 Use composition of functions to show that $f^{-1}(x) = \frac{1}{2}x - 2$ is the inverse function of $f(x) = 2x + 4$.

Solution *See page S16*

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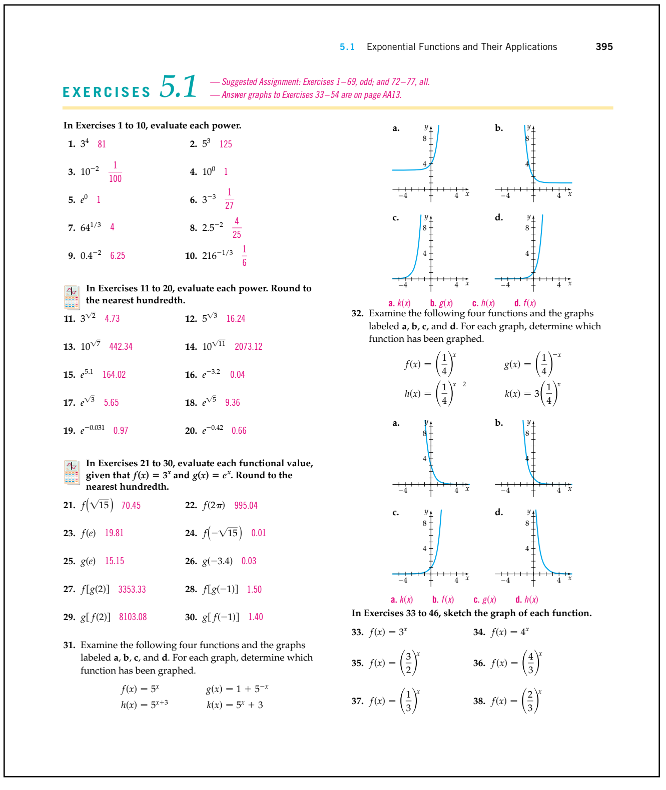

5.1 Exponential Functions and Their Applications 395

EXERCISES 5.1
— Suggested Assignment: Exercises 1–69, odd; and 72–77, all.
— Answer graphs to Exercises 33–54 are on page AA13.

In Exercises 1 to 10, evaluate each power.

1. 3^4 81
2. 5^3 125
3. 10^{-2} $\frac{1}{100}$
4. 10^0 1
5. e^0 1
6. 3^{-3} $\frac{1}{27}$
7. $64^{1/3}$ 4
8. 2.5^{-2} $\frac{4}{25}$
9. 0.4^{-2} 6.25
10. $216^{-1/3}$ $\frac{1}{6}$

In Exercises 11 to 20, evaluate each power. Round to the nearest hundredth.

11. $3^{\sqrt{2}}$ 4.73
12. $5^{\sqrt{3}}$ 16.24
13. $10^{\sqrt{7}}$ 442.34
14. $10^{\sqrt{11}}$ 2073.12
15. $e^{5.1}$ 164.02
16. $e^{-3.2}$ 0.04
17. $e^{\sqrt{3}}$ 5.65
18. $e^{\sqrt{5}}$ 9.36
19. $e^{-0.031}$ 0.97
20. $e^{-0.42}$ 0.66

In Exercises 21 to 30, evaluate each functional value, given that $f(x) = 3^x$ and $g(x) = e^x$. Round to the nearest hundredth.

21. $f(\sqrt{15})$ 70.45
22. $f(2\pi)$ 995.04
23. $f(e)$ 19.81
24. $f(-\sqrt{15})$ 0.01
25. $g(e)$ 15.15
26. $g(-3.4)$ 0.03
27. $f[g(2)]$ 3353.33
28. $f[g(-1)]$ 1.50
29. $g[f(2)]$ 8103.08
30. $g[f(-1)]$ 1.40

31. Examine the following four functions and the graphs labeled **a**, **b**, **c**, and **d**. For each graph, determine which function has been graphed.

$$f(x) = 5^x \qquad g(x) = 1 + 5^{-x}$$
$$h(x) = 5^{x+3} \qquad k(x) = 5^x + 3$$

a. $k(x)$ b. $g(x)$ c. $h(x)$ d. $f(x)$

32. Examine the following four functions and the graphs labeled **a**, **b**, **c**, and **d**. For each graph, determine which function has been graphed.

$$f(x) = \left(\frac{1}{4}\right)^x \qquad g(x) = \left(\frac{1}{4}\right)^{-x}$$
$$h(x) = \left(\frac{1}{4}\right)^{x-2} \qquad k(x) = 3\left(\frac{1}{4}\right)^x$$

a. $k(x)$ b. $f(x)$ c. $g(x)$ d. $h(x)$

In Exercises 33 to 46, sketch the graph of each function.

33. $f(x) = 3^x$
34. $f(x) = 4^x$
35. $f(x) = \left(\frac{3}{2}\right)^x$
36. $f(x) = \left(\frac{4}{3}\right)^x$
37. $f(x) = \left(\frac{1}{3}\right)^x$
38. $f(x) = \left(\frac{2}{3}\right)^x$

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A **Suggested Assignment** is provided for each section. Answers for all exercises are provided.