Chapter 8 Repetitive Change: Cycles and Trigonometry

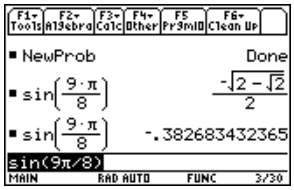
8.1 Functions of Angles: Sine and Cosine

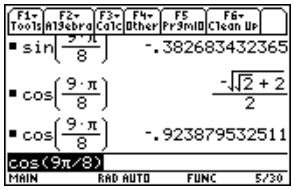
Before you begin this chapter, go back to the first page of this *Guide* and check the basic set-up and the graph format. If these are not set as specified in Figures 1–4, you will have trouble using your calculator in this chapter. Pay careful attention to the fourth line on page 1 of the MODE screen in the basic setup. The Radian/Degree mode setting affects the TI-89's interpretation of the ANGLE menu choices. The calculator's MODE menu should always be set to use Radian unless otherwise specified. (Note that calculator instructions for the material that is in the *Trigonometry Appendix* at the *Calculus Concepts* web site is on TI89-104 of this *Guide*.)

8.1.1 FINDING OUTPUTS OF TRIG FUNCTIONS WITH RADIAN INPUTS It is essential that you have the correct mode set when evaluating trigonometric function outputs. The angle setting in the MODE menu must be Radian for all applications in Chapter 8. We show how to evaluate trig functions as presented in Example 2 of Section 8.1 of *Calculus Concepts*:

Find $\sin\frac{9\pi}{8}$ and $\cos\frac{9\pi}{8}$. Because these angles are in radians, be certain that Radian is chosen in the fourth line on the first page of the MODE screen.

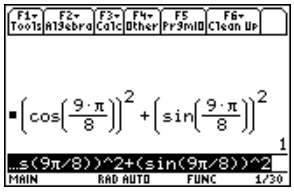
The sin and cos keys are the second function of the [Y] and [Z] keys on the TI-89 keyboard. Remember that [◆] [ENTER] gives a decimal approximation to the last answer.





It is also essential that you use parentheses to indicate the order of operations. When you press any of the trig function keys, the left parenthesis automatically appears and cannot be deleted. If anything follows the angle, the right parenthesis is necessary to show the end of the input of the trig function.

Because the cosine and the sine values are, respectively, the *x*- and *y*- coordinates of the point where the terminal side of the angle $\frac{9\pi}{8}$ intersects the unit circle, these values satisfy the equation $x^2 + y^2 = 1$. The fact that $(\cos\theta)^2 + (\sin\theta)^2 = 1$ is true for any angle θ.



8.2 Cyclic Functions as Models

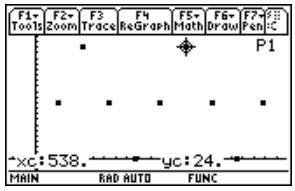
We now introduce another model – the sine model. As you might expect, this function should be fit to data that repeatedly varies between alternate extremes. The form of the sine model is given by $f(x) = a \sin(bx + h) + k$ where $|a|$ is the amplitude, b is the frequency (where $b > 0$), $2\pi/b$ is the period, $|h|/b$ is the horizontal shift (to the right if $h < 0$ and to the left if $h > 0$), and k is the vertical shift (up if $k > 0$ and down if $k < 0$). *Note:* The TI-89 uses the *c* when we use *h* and *d* when we use *k*.

8.2.1 FITTING A SINE MODEL TO DATA Before fitting any model to data, remember that you should construct a scatter plot of the data and observe what pattern the data appear to follow. Example 2 in Chapter 8 asks you to find a sine model for cyclic data with the hours of daylight on the Arctic Circle as a function of the day of the year on which the hours of daylight are measured. (January 1 is day 1.) These data appear in Table 8.3 of *Calculus Concepts*.

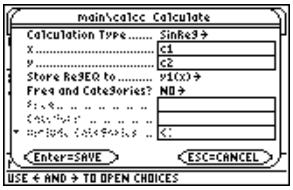
Day of the year	−10	81.5	173	264	355	446.5	538	629	720	811.5
Hours of daylight	0	12	24	12	0	12	24	12	0	12

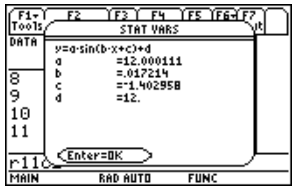
Clear any old data. Delete any functions in the Y= list and turn on Plot 1. Enter the data in the above table in lists c1 and c2. Construct a scatter plot of the data. When using the sine regression in the TI-89, it is sometimes necessary to have an estimate of the period of the data.

The data appear to be cyclic. Either look at the data or TRACE the scatter plot to measure the horizontal distance between one high point and the next (or between any two successive low points). One cycle of the data appears to be about 538 – 173 = 365 days.



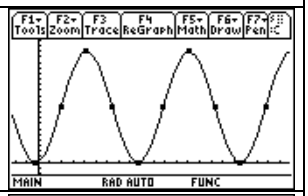
From within the data editor, follow the same procedure that you did to fit other functions: press [F5] [Calc] [▶] and choose [alpha] B [SinReg] on the first line, type in c1 for x and c2 for y, and on the fourth line, choose y1(x). Press [ENTER].





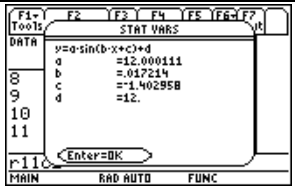
Graph the model on the scatter plot of the data.

(If the graph of the function looks like a line, you have not set the MODE menu to Radians!)



It did not occur in this example, but you might get a *Singular Matrix* error when trying to fit a sine model to data. If so, try specifying an estimate for the number of iterations[1] and the period of the function. Recall that our estimate of the period is 365 days.

Choose YES in the *Freq and Categories* line.



For these data, the same function results.

NOTE: If you do not think the function the calculator finds fits the data very well, try specifying a period and see if a better-fitting function results. It didn't here, but it might with a different set of data. If you do not specify a period, the input values should be in sequential order and should be evenly spaced. An error message and/or poor-fitting function did not result here with input that is not evenly spaced, but it may with another set of data. If so, specifying the number of iterations and a period should produce a function.

[1] An *iteration* is one repetition of the routine the TI-89 uses to fit the sine function. The number of iterations can be between 1 and 16. A large the number of iterations can produce a much better fitting function but it will also take the TI-89 longer to find the function. If the number of iterations is not specified, it defaults to 8.

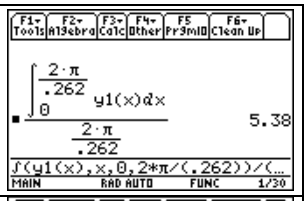
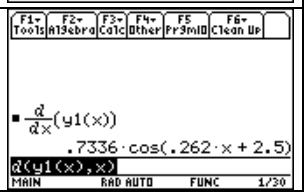
8.3 Rates of Change and Derivatives

All the previous techniques given for other functions also hold for the sine model. You can find intersections, maxima, minima, inflection points, derivatives, integrals, and so forth.

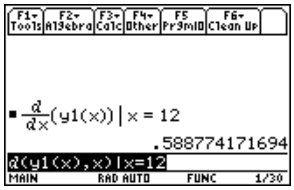
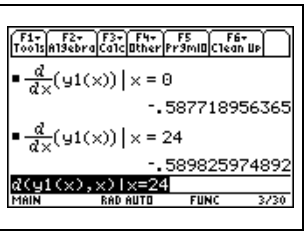
8.2.1 DERIVATIVES OF SINE AND COSINE FUNCTIONS Either evaluate the calculator's derivative or integral at a particular input or find the formula algebraically using the symbolic capabilities and evaluate it. We illustrate with Example 1 in Section 8.3 of *Calculus Concepts*:

> The calls for service made to a county sheriff's department in a certain rural/suburban county can be modeled as $c(t) = 2.8 \sin(0.262t + 2.5) + 5.38$ calls during the tth hour after midnight.

Part *a* of this example asks for the average number of calls the county sheriff's department receives each hour. The easiest way to obtain this answer is to remember that the parameter k (d in the TI-89) in the sine function is the average value.

You can also find the average value over one period of the function using the methods of Section 7.4.1. Enter the function c in y1 and type in the quotient shown to the right.	Average Value = $\dfrac{\int_0^{2\pi/0.262} c(t)\,dt}{2\pi/0.262 - 0}$	
The answer to part *b* is the formula for $c'(t)$. (Note that in part *a* or this part of the problem, the function c can be entered directly on the home screen instead of in y1.)		

Part *c* of Example 1 that asks how quickly the number of calls received each hour is changing at noon and at midnight. (Refer to Section 4.2.1 for a review of the TI-89 derivative.)

To answer these questions, simply evaluate your derivative or the TI-89's derivative at 12 for noon and 0 (or 24) for midnight. (You were not told if "midnight" refers to the initial time or 24 hours after that initial time.)		

8.5 Accumulation in Cycles

As with the other functions we have studied, applications of accumulated change with the sine and cosine functions involve the calculator's integral instruction.

8.5.1 INTEGRALS OF SINE AND COSINE FUNCTIONS We illustrate with Example 1 in Section 8.5 of *Calculus Concepts* that uses the rate of change of temperature in Philadelphia on August 27, 1993: $t(x) = 2.733 \cos(0.285x - 2.93)$ ºF per hour, x hours after midnight.

Find the accumulated change in the temperature between 9 a.m. and 3 p.m. using the TI-89 integral. The temperature increased by about 13ºF between 9 a.m. and 3 p.m.