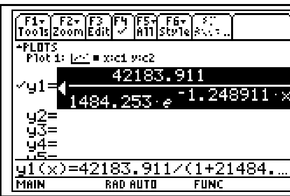
Chapter 4 Determining Change: Derivatives

4.1 Numerically Finding Slopes

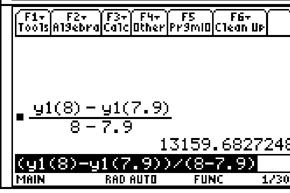
Using your calculator to find slopes of tangent lines does not involve a new procedure. However, the techniques that are discussed in this section allow you to repeatedly apply a method of finding slopes that gives quick and accurate results.

4.1.1 NUMERICALLY ESTIMATING SLOPES ON THE HOME SCREEN Finding the slopes of secant lines joining the point at which the tangent line is drawn to increasingly close points on a function to the left and right of the point of tangency is easily done using your TI-89. Suppose we want to numerically estimate the slope of the tangent line at $t = 8$ to the graph of the function that gives the number of polio cases in 1949: $y = \dfrac{42{,}183.911}{1 + 21{,}484.253e^{-1.248911t}}$ where $t = 1$ on January 31, 1949, $t = 2$ on February 28, 1949, and so forth.

Enter the polio cases equation in the y1 location of the Y= list. (Carefully check your entry of the equation, and be sure to use parentheses around the denominator and the exponent. Use x as the input variable.) We now evaluate the slopes of secant lines that join close points to the left of $x = 8$ with $x = 8$.

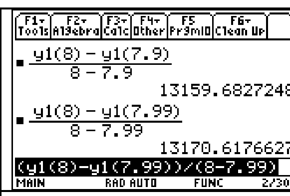


On the home screen, type in the expression shown to the right to compute the slope of the secant line joining the close point where $x = 7.9$ and the point of tangency where $x = 8$.



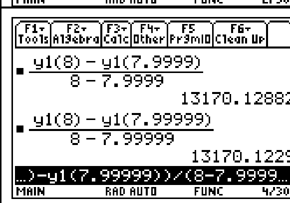
Record on paper each slope, to at least 1 more decimal place than the desired accuracy, as it is computed. You are asked to find the nearest whole number that these slopes are approaching, so record at least one decimal place in your table of slopes.

Press [▶] to prepare to edit the entry line, and then use [◀] to move the cursor behind each 9 in "7.9". Press 9 to insert another 9 in <u>both</u> places that 7.9 appears. Press [ENTER] to find the slope of the secant line joining $x = 7.99$ and $x = 8$.



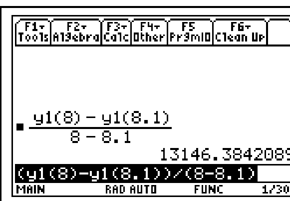
Continue in this manner, recording each result on paper, until you can determine to which value the slopes from the left seem to be getting closer and closer.

It appears that the slopes of the secant lines from the left are approaching 13,170 cases per month.



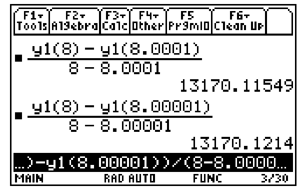
We now find slopes joining the point of tangency and nearby close points to the *right* of $x = 8$.

Use [▲] until you find the instruction containing "7.9". Press [ENTER] to copy this instruction to the entry line. Now edit the entry line so that the nearby point is $x = 8.1$, not 7.9. Press [ENTER] to calculate the secant line slope.



Continue in this manner as you did when calculating slopes to the left, but each time insert a 0 before the "1" in two places in the close point. Record each result on paper until you can determine the value the slopes from the right are approaching.

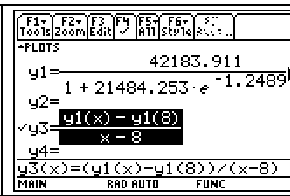
It appears that the slopes of the secant lines from the right are approaching 13,170 cases per month.



When the slopes from the left and the slopes from the right approach the same number, that number is the slope of the tangent line at the point of tangency. In this case, we estimate the slope of the tangent line to be 13,170 cases per month.

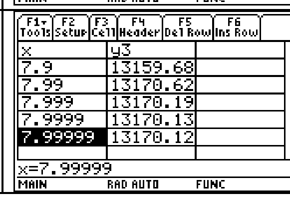
4.1.2 NUMERICALLY ESTIMATING SLOPES USING THE TABLE The process discussed in Section 4.1.1 can be done in fewer steps and with fewer keystrokes when you use the TI-89 table. The point of tangency is $x = 8$, $y =$ y1(8), and let's call the close point (x, y1(x)). Then, slope = $\frac{\text{rise}}{\text{run}} = \frac{\text{y1(x)} - \text{y1(8)}}{\text{x} - 8}$. We illustrate numerically estimating the slope using the table and the logistic function given in Section 4.1.1 of this *Guide*.

Have the polio cases equation given in the previous section in the y1 location of the Y= list. Enter the above slope formula in a different other location, say y3. (Remember to enclose both numerator and denominator of the slope formula in parentheses.) Turn off y1 because we are only considering the y3 output.



Press ◆ F4 (TblSet); choose ASK in the Independent: location.

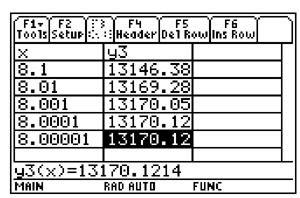
Access the table with ◆ F5 (TABLE) and clear any previous entries. Enter values for x, the input of the close point, so that x gets closer and closer to 8 from the left.



- Although it did not happen in this illustration, the calculator might switch the input values you enter to scientific notation and display rounded output values so that the numbers can fit on the screen in the space allotted for outputs of the table. If this happens, you should position the cursor over each output value and record on paper as many decimal places as necessary in order to determine the limit to the desired degree of accuracy.

Repeat the process, entering values for x, the input of the close point, so that x gets closer and closer to 8 from the right.

Remember to highlight the output values and record on paper the slope to at least one more decimal position than the desired accuracy.



The limit is estimated to be the value that the limits from the left and the right are approaching: 13,170 cases per month.

NOTE: You may wish to leave the slope formula in y3 as long as you need it. Remember to change the input value at the point of tangency the next time you use this formula. Turn y3 off when you are not using it by removing the check mark by the name with F4 [✓].

4.2 Algebraically Finding Slopes

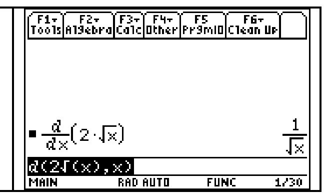
The TI-89 finds algebraic formulas for slope, and you can use the built-in derivative function to draw the graph of a derivative and to check any formula that you find algebraically.

4.2.1 UNDERSTANDING YOUR CALCULATOR'S SLOPE FUNCTIONS The TI-89 has two derivative (slope) instructions: a numeric derivative[1] called nDeriv and what we think of as the mathematical derivative *d*(). Because of its symbolic capabilities and accuracy, we use *d*() in this *Guide*. The TI-89 syntax[2] for this instruction is *d(function, input variable)*. We illustrate the use of this slope function in the next section of this *Guide*.

WARNING: If you have a number stored to what you are using as the input variable, a numerical result rather than a symbolic one, is given by the TI-89. It is a good idea from this point on to follow the instructions given at the beginning of this *Guide* and begin each new problem by pressing [HOME] [2nd] [F1] [F6: Clean Up] 2 [NewProb] [ENTER] or [HOME] [2nd] [F1] [F6: Clean Up] 1 [Clear a-z…] [ENTER].

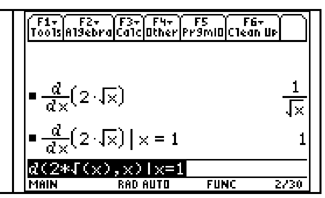
Any smooth, continuous function will do for our explorations, so let's investigate with the function in Example 3 of Section 4.2 of *Calculus Concepts*: $f(x) = 2\sqrt{x}$.

On the home screen, access the derivative instruction with [2nd] 8 (*d*). Next enter the function and the input variable, separated by a comma. The derivative formula (that is, the symbolic derivative) is displayed.



- If your result is a number, not a formula in terms of *x*, see the warning note above.

To evaluate the derivative at a particular input, enter that input following the "with" operator, which prints as "|". Access this operator by pressing the key directly under [=].



NOTE: The "with" operator is very versatile. It can be used at the end of an instruction for evaluating many different functions and expressions, for specifying conditions with a solve instruction, and so forth. As seen in Chapter 1 of this *Guide*, it is used to temporarily override a variable's defined value or to temporarily define a value for an undefined variable.

4.4 More Simple Rate-of-Change Formulas

The TI-89 calculates numerical values of slopes as well as giving the slope in formula form. We first look at the correspondence between the mathematical notation and TI-89 notation.

4.4.1 DERIVATIVE NOTATION AND CALCULATOR NOTATION In addition to learning the derivative formulas and how to use them so that you will know when your calculator gives

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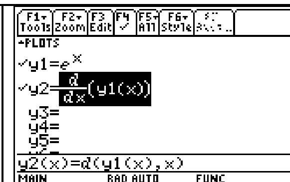
[1] If you want more information on the numeric derivative, refer to page 446 of the *TI-89 Guidebook* and Section 4.2.1 of the *TI-83 Graphing Calculator Instruction Guide* at the beginning of this *Guide*.

[2] The complete syntax is *d(function, input variable, order)*. The *order* is the number of the derivative you are trying to find: 1 for first derivative, 2 for second derivative, and so forth. More will be explained about the order in Chapter 5.

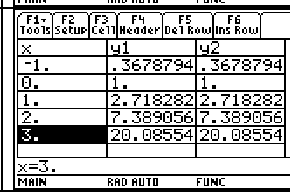
an acceptable answer for a derivative and when it does not, you also need to understand the differences and similarities in mathematical derivative notation and calculator notation. The notation that we use for the calculator's derivative is *d(f(x), x)*. When you enter this expression for a particular function, it prints in the history area almost the same as one of the ways we write it mathematically: We write $\frac{df(x)}{dx}$ or $f'(x)$ and the TI-89 prints $\frac{d}{dx}(f(x))$.

We illustrate a use of the TI-89's derivative by constructing Table 4.14 in Section 4.4 of *Calculus Concepts*. The table lists x, $y = f(x) = e^x$, and $y' = \frac{df}{dx}$ for seven different inputs.

You can evaluate the TI-89 derivative on the home screen or in the table. We choose to use the table because we are evaluating at several values. Enter $f(x) = e^x$ in the Y= list, say in y1. In y2 enter the derivative evaluated at a general input x.



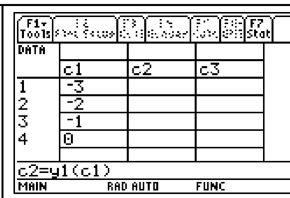
Press [◆] [F4] (TblSet), and choose ASK in the Independent: location. Access the table with [◆] [F5] (TABLE) and enter −3, −2, −1, 0, 1, 2, and 3 for x.



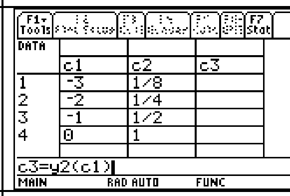
It appears that the derivative values are the same as the function outputs. In fact, this is a true statement for all inputs of *f* – this function is its own derivative!

Press [◆] [F1] (Y=) and edit y1 to be the function $g(x) = 2^x$.

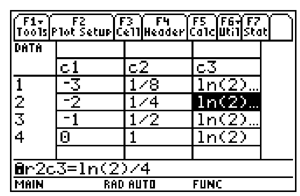
Access the statistical lists and clear any previous entries from c1, c2, c3, and c4. Enter the *x*-values enter −3, −2, −1, 0, 1, 2, and 3 in c1. Highlight any cell in c2, press [F4] [Header], and type* y1(c1).



Press [ENTER] to fill c2 with the function outputs. Then, highlight any cell in c3, press [F4] [Header], and type* y2(c1).



Press [ENTER] to place in list c3 the derivative of y1 evaluated at the inputs in c1. Note that these values are not the same as the function outputs. However press [▼], look at the bottom of the screen, and see that each value in c3 is ln 2 times the corresponding value in c2.

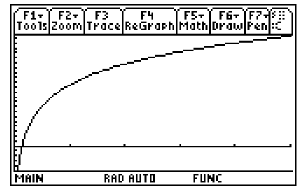
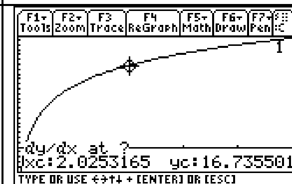
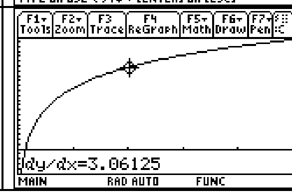


*Remember to delete the column instruction (after the values have been filled in as directed) by pressing [F4] [Header] and [CLEAR]. [ENTER] returns the cursor to the table.

This investigation strongly suggests that the slope formula is: If $g(x) = 2^x$, then $\frac{dg}{dx} = (\ln 2)\, 2^x$.

4.4.2 CALCULATING $\frac{dy}{dx}$ AT SPECIFIC INPUT VALUES In the previous two sections of this *Guide*, we entered and evaluated the derivative instruction on the home screen and in the Y= list and the table. You can also evaluate the TI-89 derivative from the graphics screen with the

Math menu. We illustrate this with the function in part *a* of Example 2 in Section 4.4 of *Calculus Concepts*.

Enter $f(x) = 12.36 + 6.2 \ln x$ in y1. Access LN with [2nd] [X]. (All previously-entered functions should be cleared or turned off except for this one in y1.) We want to draw a graph of *f*. Realize that because of the log term, $x \geq 0$. Choose some value for xmax, say 5. Use [F2] [Zoom] [alpha] [=] (A) [ZoomFit] to set the height of the graph.	
With the graph on the screen, press [F5] [Math] 6 [Derivatives] 1 [dy/dx]. Use [◄] or [►] to move to some point on the graph or type in an input value at which you want to find the derivative.	
Press [ENTER] and the slope of the function is calculated at the input you chose in the previous step.	

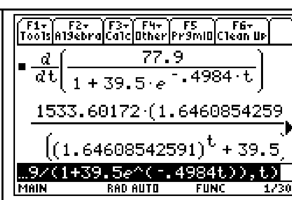
4.4.3 NUMERICALLY CHECKING SLOPE FORMULAS
4.4.4 GRAPHICALLY CHECKING SLOPE FORMULAS
4.5.1 SUMMARY OF CHECKING METHODS

Before you begin checking your answer, make sure that you have correctly entered the function. It is very frustrating to miss the answer to a problem because you have made an error in entering a function in your calculator. Also carefully compare the expression you typed (once you have entered it on the home screen or in the graphing list) to what you have written on your paper or with what appears in the textbook and make certain that they appear identical.

It is always a good idea to check your answer. Because the TI-89 gives the algebraic formula for a derivative function, you can check your answer simply by looking at the calculator screen. You therefore do not need to use numerical techniques to check your answer, so these three sections of the *Guide* are unnecessary.

4.5 The Chain Rule

We finish illustrating using the derivative function using the function in Example 3 of Section 4.5 of *Calculus Concepts*. (These ideas also apply to Section 4.6.)

On the home screen, press [2nd] 8 (*d*), type $\frac{77.9}{1+39.5e^{-0.4984t}}$, press [,] [T] [)] [ENTER]. (Note that the result is given in terms of b^t, not e^{at}.)	
Evaluate the derivative at $t = 5$ by pressing [►] and edit the previous entry by typing "| t = 5". Press [ENTER].