C H A P T E R 4
Rational Expressions, Equations, and Functions

Section 4.1 Integer Exponents and Scientific Notation

Solutions to Odd-Numbered Exercises

1. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

3. $-10^{-3} = -\frac{1}{10^3} = -\frac{1}{1000}$

5. $(-3)^0 = 1$

7. $\frac{1}{4^{-3}} = \frac{1}{(1/4)^3} = 4^3 = 64$

9. $\frac{1}{(-2)^{-5}} = \frac{1}{\left(-\frac{1}{2}\right)^5}$

$= \frac{1}{-\frac{1}{32}} = -32$

11. $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

13. $\left(\frac{3}{16}\right)^0 = 1$

15. $27 \cdot 3^{-3} = 3^3 \cdot 3^{-3}$

$= 3^{3+(-3)}$

$= 3^0$

$= 1$

17. $\frac{3^4}{3^{-2}} = 3^{4-(-2)}$

$= 3^{4+2}$

$= 3^6$

$= 729$

19. $\frac{10^3}{10^{-2}} = 10^{3-(-2)}$

$= 10^{3+2}$

$= 10^5$

$= 100{,}000$

21. $(4^2 \cdot 4^{-1})^{-2} = (4^{2+(-1)})^{-2}$

$= (4^1)^{-2}$

$= 4^{-2}$

$= \frac{1}{4^2}$

$= \frac{1}{16}$

23. $(2^{-3})^2 = 2^{-6}$

$= \frac{1}{2^6}$

$= \frac{1}{64}$

25. $2^{-3} + 2^{-4} = \frac{1}{2^3} + \frac{1}{2^4}$

$= \frac{1}{8} + \frac{1}{16}$

$= \frac{2}{16} + \frac{1}{16}$

$= \frac{3}{16}$

27. $\left(\frac{3}{4} + \frac{5}{8}\right)^{-2} = \left(\frac{6}{8} + \frac{5}{8}\right)^{-2}$

$= \left(\frac{11}{8}\right)^{-2}$

$= \left(\frac{8}{11}\right)^2$

$= \frac{64}{121}$

29. $(5^0 - 4^{-2})^{-1} = \left(1 - \frac{1}{4^2}\right)^{-1}$

$= \left(\frac{16}{16} - \frac{1}{16}\right)^{-1}$

$= \left(\frac{15}{16}\right)^{-1}$

$= \frac{16}{15}$

31. $y^4 \cdot y^{-2} = y^{4+(-2)} = y^2$

33. $z^5 \cdot z^{-3} = z^{5+(-3)} = z^2$

35. $7x^{-4} = \frac{7}{x^4}$

37. $(4x)^{-3} = \dfrac{1}{(4x)^3} = \dfrac{1}{64x^3}$

39. $\dfrac{1}{x^{-6}} = x^6$

41. $\dfrac{8a^{-6}}{6a^{-7}} = \dfrac{4}{3}a^{(-6)-(-7)}$
$= \dfrac{4}{3}a^{-6+7}$
$= \dfrac{4}{3}a$

43. $\dfrac{(4t)^0}{t^{-2}} = \dfrac{1}{t^{-2}} = t^2$

45. $(2x^2)^{-2} = \dfrac{1}{(2x^2)^2} = \dfrac{1}{4x^4}$

47. $(-3x^{-3}y^2)(4x^2y^{-5}) = -3 \cdot 4 \cdot x^{-3+2} \cdot y^{2+(-5)}$
$= -12x^{-1}y^{-3}$
$= -\dfrac{12}{xy^3}$

49. $(3x^2y^{-2}) = 3^{-2}x^{-4}y^4 = \dfrac{1y^4}{9x^4}$

51. $\left(\dfrac{x}{10}\right)^{-1} = \dfrac{10}{x}$

53. $\dfrac{6x^3y^{-3}}{12x^{-2}y} = \dfrac{6x^{3-(-2)}y^{-3-1}}{6 \cdot 2}$
$= \dfrac{x^5y^{-4}}{2}$
$= \dfrac{x^5}{2y^4}$

55. $\left(\dfrac{3u^2v^{-1}}{3^3u^{-1}v^3}\right)^{-2} = \left(\dfrac{3u^{2-(-1)}v^{-1-3}}{3^3}\right)^{-2}$
$= \left(\dfrac{u^3v^{-4}}{3^2}\right)^{-2}$
$= \left(\dfrac{3^2}{u^3v^{-4}}\right)^{2}$
$= \dfrac{3^4}{u^6v^{-8}}$
$= \dfrac{81v^8}{u^6}$

57. $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3 = \left(\dfrac{b^2}{a^2}\right)\left(\dfrac{b^3}{a^3}\right)$
$= \dfrac{b^5}{a^5}$

59. $(2x^3y^{-1})^{-3}(4xy^{-6}) = (2^{-3}x^{-9}y^3)(4xy^{-6})$
$= \dfrac{4x^{-9+1}y^{3+(-6)}}{2^3}$
$= \dfrac{4x^{-8}y^{-3}}{8}$
$= \dfrac{1}{2x^8y^3}$

61. $u^4(6u^{-3}v^0)(7v)^0 = u^4(6u^{-3})(1)$
$= 6u^{4+(-3)}$
$= 6u$

63. $[(x^{-4}y^{-6})^{-1}]^2 = (x^4y^6)^2 = x^8y^{12}$

65. $\dfrac{(2a^{-2}b^4)^3b}{(10a^3b)^2} = \dfrac{2^3a^{-6}b^{12} \cdot b}{10^2a^6b^2}$
$= \dfrac{8a^{-6-6}b^{12+1-2}}{100}$
$= \dfrac{2a^{-12}b^{11}}{25}$
$= \dfrac{2b^{11}}{25a^{12}}$

67. $(u + v^{-2})^{-1} = \dfrac{1}{u + v^{-2}}$
$= \dfrac{1}{u + \left(\dfrac{1}{v^2}\right)} \cdot \dfrac{v^2}{v^2}$
$= \dfrac{v^2}{uv^2 + 1}$

69. $\dfrac{a+b}{ba^{-1}-ab^{-1}} = \dfrac{a+b}{\frac{b}{a}-\frac{a}{b}} \cdot \dfrac{ab}{ab}$

$= \dfrac{a^2b+ab^2}{b^2-a^2}$

$= \dfrac{ab(a+b)}{(b-a)(b+a)}$

$= \dfrac{ab}{b-a}$

71. $3{,}600{,}000 = 3.6 \times 10^6$

73. $47{,}620{,}000 = 4.762 \times 10^7$

75. $0.00031 = 3.1 \times 10^{-4}$

77. $0.0000000381 = 3.81 \times 10^{-8}$

79. $57{,}500{,}000 = 5.75 \times 10^7$

81. $9{,}461{,}000{,}000{,}000{,}000 = 9.461 \times 10^{15}$

83. $0.0000899 = 8.99 \times 10^{-5}$

85. $6 \times 10^7 = 60{,}000{,}000$

87. $1.359 \times 10^{-7} = 0.0000001359$

89. $\$3.17 \times 10^{10} = 31{,}700{,}000{,}000$

91. $1.3 \times 10^7 = 13{,}000{,}000$

93. $4.8 \times 10^{-10} = 0.00000000048$

95. $(2 \times 10^9)(3.4 \times 10^{-4}) = (2)(3.4)(10^5)$

$= 6.8 \times 10^5$

97. $(5 \times 10^4)^2 = 5^2 \times 10^8$

$= 25 \times 10^8$

$= 2.5 \times 10^9$

99. $\dfrac{3.6 \times 10^{12}}{6 \times 10^5} = \dfrac{3.6}{6} \times 10^{12-5}$

$= 0.6 \times 10^7$

$= 6.0 \times 10^6$

101. $(4{,}500{,}000)(2{,}000{,}000{,}000) = (4.5 \times 10^6)(2 \times 10^9)$

$= (4.5)(2) \times 10^{15}$

$= 9 \times 10^{15}$

103. $\dfrac{64{,}000{,}000}{0.00004} = \dfrac{6.4 \times 10^7}{4.0 \times 10^{-5}}$

$= 1.6 \times 10^{7-(-5)}$

$= 1.6 \times 10^{12}$

105. $\dfrac{(0.0000565)(2{,}850{,}000{,}000{,}000)}{0.00465} = \dfrac{(5.65 \times 10^{-5})(2.85 \times 10^{12})}{4.65 \times 10^{-3}}$

$= \dfrac{(5.65)(2.85)}{4.65} \times 10^{10}$

$\approx 3.4629032 \times 10^{10}$

$\approx 3.46 \times 10^{10}$

107. $\dfrac{1.357 \times 10^{12}}{(4.2 \times 10^2)(6.87 \times 10^{-3})} = \dfrac{1.357}{(4.2)(6.87)} \times 10^{13}$

$= 0.0470299 \times 10^{13}$

$= 4.70299 \times 10^{11}$

$\approx 4.70 \times 10^{11}$

109. $72{,}400 \times 2{,}300{,}000{,}000 = (7.24 \times 10^4)(2.3 \times 10^9)$

$= 16.652 \times 10^{4+9}$

$= 16.652 \times 10^{13}$

$= 1.6652 \times 10^{14}$

$\approx 1.67 \times 10^{14}$

111. $\dfrac{(5{,}000{,}000)^3(0.000037)^2}{(0.005)^4} = \dfrac{(5.0 \times 10^6)^3(3.7 \times 10^{-5})^2}{(5.0 \times 10^{-3})^4}$

$= \dfrac{(5^3 \times 10^{18})(3.7^2 \times 10^{-10})}{5^4 \times 10^{-12}}$

$= \dfrac{(125)(13.69)}{625} \times 10^{18+(-10)-(-12)}$

$= 2.738 \times 10^{18-10+12}$

$= 2.738 \times 10^{20}$

$\approx 2.74 \times 10^{20}$

113. $93{,}000{,}000 = 9.3 \times 10^7$

115. $\dfrac{1.49 \times 10^{11}}{9.45 \times 10^{15}} = \dfrac{1.49}{9.45} \times 10^{-4}$

$\approx 0.157672 \times 10^{-4}$

$\approx 1.58 \times 10^{-5}$

≈ 8.3 minutes

117. $\dfrac{1.99 \times 10^{30}}{5.975 \times 10^{24}} = \dfrac{1.99}{5.975} \times 10^6$

$\approx 0.3330544 \times 10^6$

$\approx 3.33 \times 10^5$

119. $\dfrac{\$5506 \text{ billion}}{270 \text{ million}} = \dfrac{\$5{,}506{,}000{,}000{,}000}{270{,}000{,}000}$

$= \dfrac{5.506 \times 10^{12}}{2.7 \times 10^8}$

$\approx 2.03925 \times 10^4$

$\approx \$20{,}393$

121. In $(3x)^4$, $3x$ is called the base and 4 is called the exponent.

123. You can "move" a factor from the numerator to the denominator by changing the sign of the exponent of the factor.

125. Scientific notation is an efficient way of writing and computing real numbers when the numbers are very large or very small.

Section 4.2 Rational Expressions and Functions

1. $x - 8 \neq 0$

$x \neq 8$

$D = (-\infty, 8) \cup (8, \infty)$

3. $x + 4 \neq 0$

$x \neq -4$

$D = (-\infty, -4) \cup (-4, \infty)$

5. $4 \neq 0$

$D = (-\infty, \infty)$

7. $D = (-\infty, \infty)$

9. $x^2 + 4 \neq 0$

$D = (-\infty, \infty)$

11. $y(y + 3) \neq 0$

$y \neq 0 \quad y \neq -3$

$D = (-\infty, -3) \cup (-3, 0) \cup (0, \infty)$