C H A P T E R 3
Polynomials and Factoring

Section 3.1 Adding and Subtracting Polynomials

Solutions to Odd-Numbered Exercises

1. Standard form: $10x - 4$

Degree: 1

Leading coefficient: 10

3. Standard form: $3x^2 - x + 2$

Degree: 2

Leading coefficient: 3

5. Standard form: $y^5 - 3y^4 - 2y^3 + 5$

Degree: 5

Leading coefficient: 1

7. Standard form: $-3x^3 - 2x^2 - 3$

Degree: 3

Leading coefficient: -3

9. Standard form: -4

Degree: 0

Leading coefficient: -4

11. Standard form: $-16t^2 + v_0t$

Degree: 2

Leading coefficient: -16

13. $12 - 5y^2$ is a binomial.

15. $x^3 + 2x^2 - 4$ is a trinomial.

17. 5 is a monomial.

19. A monomial of degree 3 is any term of form ax^3 where a is any real number.

21. A binomial of degree 2 and leading coefficient of 8 is any binomial beginning $8x^2$ and containing one other term of degree less than 2 such as $8x^2 + 4$ or $8x^2 + x$.

23. $y^{-3} - 2$ is not a polynomial because the first term is not of the form ax^k (k must be nonnegative).

25. $\frac{8}{x}$ is not a polynomial because the term is not of the form ax^k (k must be nonnegative).

27. $5 + (2 + 3x) = (5 + 2) + 3x = 7 + 3x$

29. $(2x^2 - 3) + (5x^2 + 6) = (2x^2 + 5x^2) + (-3 + 6) = 7x^2 + 3$

31. $(5y + 6) + (4y^2 - 6y - 3) = 4y^2 + (5y - 6y) + (6 - 3) = 4y^2 - y + 3$

33. $(2 - 8y) + (-2y^4 + 3y + 2) = (-2y^4) + (-8y + 3y) + (2 + 2) = -2y^4 - 5y + 4$

35. $(8 - t^4) + (5 + t^4) = (8 + 5) + (-t^4 + t^4) = 13$

37. $(x^2 - 3x + 8) + (2x^2 - 4x) + 3x^2 = (x^2 + 2x^2 + 3x^2) + (-3x - 4x) + (8) = 6x^2 - 7x + 8$

39. $\left(\frac{2}{3}x^3 - 4x + 1\right) + \left(-\frac{3}{5} + 7x - \frac{1}{2}x^3\right) = \left(\frac{2}{3}x^3 - \frac{1}{2}x^3\right) + (-4x + 7x) + \left(1 - \frac{3}{5}\right) = \left(\frac{4}{6}x^3 - \frac{3}{6}x^3\right) + 3x + \left(\frac{5}{5} - \frac{3}{5}\right) = \frac{1}{6}x^3 + 3x + \frac{2}{5}$

41. $(6.32t - 4.51t^2) + (7.2t^2 + 1.03t - 4.2) = (-4.51t^2 + 7.2t^2) + (6.32t + 1.03t) - 4.2 = 2.69t^2 + 7.35t - 4.2$

43.
$$\begin{array}{r} 5x^2 - 3x + 4 \\ -3x^2 \qquad - 4 \\ \hline 2x^2 - 3x \qquad \end{array}$$

45.
$$\begin{array}{r} 4x^3 - 2x^2 + 8x \qquad \\ 4x^2 + \ x - 6 \\ \hline 4x^3 + \ 2x^2 + 9x - 6 \end{array}$$

47.
$$\begin{array}{r} 5p^2 - 4p + 2 \\ -3p^2 + 2p - 7 \\ \hline 2p^2 - 2p - 5 \end{array}$$

49.
$$\begin{array}{l} -3.6b^2 + 2.5b \\ -2.4b^2 - 3.1b + 7.1 \\ \ \ 6.6b^2 \\ \hline \ \ 0.6b^2 - 0.6b + 7.1 \end{array}$$

51.
$$\begin{aligned} (4 - y^3) - (4 + y^3) &= (4 - y^3) + (-4 - y^3) \\ &= (4 - 4) + (-y^3 - y^3) \\ &= -2y^3 \end{aligned}$$

53.
$$\begin{aligned} (3x^2 - 2x + 1) - (2x^2 + x - 1) &= (3x^2 - 2x + 1) + (-2x^2 - x + 1) \\ &= (3x^2 - 2x^2) + (-2x - x) + (1 + 1) \\ &= x^2 - 3x + 2 \end{aligned}$$

55.
$$\begin{aligned} (6t^3 - 12) - (-t^3 + t - 2) &= (6t^3 - 12) + (t^3 - t + 2) \\ &= (6t^3 + t^3) - t + (-12 + 2) \\ &= 7t^3 - t - 10 \end{aligned}$$

57.
$$\begin{aligned} \left(\tfrac{1}{4}y^2 - 5y\right) - \left(12 + 4y - \tfrac{3}{2}y^2\right) &= \left(\tfrac{1}{4}y^2 - 5y\right) + \left(-12 - 4y + \tfrac{3}{2}y^2\right) \\ &= \left(\tfrac{1}{4}y^2 + \tfrac{3}{2}y^2\right) + (-5y - 4y) - 12 \\ &= \tfrac{7}{4}y^2 - 9y - 12 \end{aligned}$$

59.
$$\begin{aligned} (10.4t^4 - 0.23t^5 + 1.3t^2) - (2.6 - 7.35t + 6.7t^2 - 9.6t^5) &= (10.4t^4 - 0.23t^5 + 1.3t^2) + (-2.6 + 7.35t - 6.7t^2 + 9.6t^5) \\ &= (-0.23t^5 + 9.6t^5) + 10.4t^4 + (1.3t^2 - 6.7t^2) + 7.35t - 2.6 \\ &= 9.37t^5 + 10.4t^4 - 5.4t^2 + 7.35t - 2.6 \end{aligned}$$

61.
$$\begin{aligned} (x^3 - 3x) - [3x^3 - (x^2 + 5x)] &= (x^3 - 3x) - [3x^3 - x^2 - 5x] \\ &= x^3 - 3x - 3x^3 + x^2 + 5x \\ &= -2x^3 + x^2 + 2x \end{aligned}$$

63.
$$\begin{array}{rcr} x^2 - x + \ \ 3 & \Longrightarrow & x^2 - \ x + 3 \\ - \qquad (x - 2) & \Longrightarrow & - \ x + 2 \\ \hline & & x^2 - 2x + 5 \end{array}$$

65.
$$\begin{array}{rcr} 25 - 15x - 2x^3 & \Longrightarrow & -2x^3 - 15x + 25 \\ -(12 - 13x + 2x^3) & \Longrightarrow & -2x^3 + 13x - 12 \\ \hline & & -4x^3 - \ \ 2x + 13 \end{array}$$

67.
$$\begin{array}{rcr} -3x^7 \qquad\qquad + 6x^4 + \ \ 4 & \Longrightarrow & -3x^7 \qquad\qquad + 6x^4 + \ \ 4 \\ -(8x^7 + 10x^5 - 2x^4 - 12) & \Longrightarrow & -8x^7 - 10x^5 + 2x^4 + 12 \\ \hline & & -11x^7 - 10x^5 + 8x^4 + 16 \end{array}$$

69.
$$\begin{aligned} -(2x^3 - 3) + (4x^3 - 2x) &= -2x^3 + 3 + 4x^3 - 2x \\ &= (-2x^3 + 4x^3) + (-2x) + (3) \\ &= 2x^3 - 2x + 3 \end{aligned}$$

71.
$$\begin{aligned} (4x^5 - 10x^3 + 6x) - (8x^5 - 3x^3 + 11) + (4x^5 + 5x^3 - x^2) &= (4x^5 - 10x^3 + 6x) + (-8x^5 + 3x^3 - 11) + (4x^5 + 5x^3 - x^2) \\ &= (4x^5 - 8x^5 + 4x^5) + (-10x^3 + 3x^3 + 5x^3) - x^2 + 6x - 11 \\ &= -2x^3 - x^2 + 6x - 11 \end{aligned}$$

73. $$\begin{aligned}(5y^2 - 2y) - [(y^2 + y) - (3y^2 - 6y + 2)] &= (5y^2 - 2y) - [(y^2 + y) + (-3y^2 + 6y - 2)]\\ &= (5y^2 - 2y) - [(y^2 - 3y^2) + (y + 6y) - 2]\\ &= (5y^2 - 2y) - [-2y^2 + 7y - 2]\\ &= (5y^2 - 2y) + (2y^2 - 7y + 2)\\ &= (5y^2 + 2y^2) + (-2y - 7y) + 2\\ &= 7y^2 - 9y + 2\end{aligned}$$

75. $$\begin{aligned}(8x^3 - 4x^2 + 3x) - [(x^3 - 4x^2 + 5) + (x - 5)] &= (8x^3 - 4x^2 + 3x) - [x^3 - 4x^2 + x]\\ &= (8x^3 - 4x^2 + 3x) + (-x^3 + 4x^2 - x)\\ &= (8x^3 - x^3) + (-4x^2 + 4x^2) + (3x - x)\\ &= 7x^3 + 2x\end{aligned}$$

77. $$\begin{aligned}3(4x^2 - 1) + (3x^3 - 7x^2 + 5) &= 12x^2 - 3 + 3x^3 - 7x^2 + 5\\ &= 3x^3 + 5x^2 + 2\end{aligned}$$

79. $$\begin{aligned}2(t^2 + 12) - 5(t^2 + 5) + 6(t^2 + 5) &= 2t^2 + 24 - 5t^2 - 25 + 6t^2 + 30\\ &= (2t^2 - 5t^2 + 6t^2) + (24 - 25 + 30)\\ &= 3t^2 + 29\end{aligned}$$

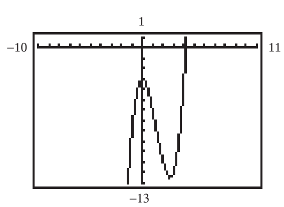
81. $$\begin{aligned}15v - 3(3v - v^2) + 9(8v + 3) &= 15v - 9v + 3v^2 + 72v + 27\\ &= (3v^2) + (15v - 9v + 72v) + 27\\ &= 3v^2 + 78v + 27\end{aligned}$$

83. $$\begin{aligned}5s - [6s - (30s + 8)] &= 5s - [6s - 30s - 8]\\ &= (5s - 6s + 30s) + (8)\\ &= 29s + 8\end{aligned}$$

85. *Keystrokes:*

y_1 [Y=] [(] [X,T,θ] [^] 3 [−] 3 [X,T,θ] [x^2] [−] 2 [)] [−] [(] [X,T,θ] [x^2] [+] 1 [)] [ENTER]

y_2 [X,T,θ] [^] 3 [−] 4 [X,T,θ] [x^2] [−] 3 [GRAPH]



y_1 and y_2 represent equivalent expressions since the graphs of y_1 and y_2 are identical.

87. $$\begin{aligned}h(x) &= f(x) + g(x)\\ &= (4x^3 - 3x^2 + 7) + (9 - x - x^2 - 5x^3)\\ &= (4x^3 - 5x^3) + (-3x^2 - x^2) - x + (7 + 9)\\ &= -x^3 - 4x^2 - x + 16\end{aligned}$$

89.

Polynomial	***Value***	***Substitute***	***Simplify***
$h(t) = -16t^2 + 64$	(a) $t = 0$	$-16(0)^2 + 64$	64 feet
	(b) $t = \frac{1}{2}$	$-16\left(\frac{1}{2}\right)^2 + 64$	60 feet
	(c) $t = 1$	$-16(1)^2 + 64$	48 feet
	(d) $t = 2$	$-16(2)^2 + 64$	0 feet

At time $t = 0$, the object is at 64 feet and continues to fall, reaching the ground at time $t = 2$.

91.

Polynomial	***Value***	***Substitute***	***Simplify***
$h(t) = -16t^2 + 80t + 50$	(a) $t = 0$	$-16(0)^2 + 80(0) + 50$	50 feet
	(b) $t = 2$	$-16(2)^2 + 80(2) + 50$	146 feet
	(c) $t = 4$	$-16(4)^2 + 80(4) + 50$	114 feet
	(d) $t = 5$	$-16(5)^2 + 80(5) + 50$	50 feet

At time $t = 0$, the object is at a height of 50 feet. The object moves upward, reaches a maximum height and returns downward. At time $t = 5$, object is again at a height of 50 feet.

93. The free-falling object was dropped.

$-16(0)^2 + 100 = 100$ feet

95. The free-falling object was thrown downward.

$-16(0)^2 - 24(0) + 50 = 50$ feet

97. $h = -16(1)^2 + 40(1) + 200 = 224$ feet

$h = -16(2)^2 + 40(2) + 200 = 216$ feet

$h = -16(3)^2 + 40(3) + 200 = 176$ feet

99. *Verbal model:* $\boxed{\text{Profit}} = \boxed{\text{Revenu}} - \boxed{\text{Cost}}$

Equation: $P = R - C$

$P = 14x - (8x + 15{,}000)$

$P = 6x - 15{,}000$

$P = 6(5000) - 15{,}000$

$P = \$15{,}000$

101. Perimeter of region $= 2(2x + 4) + 4x + 2(3x)$

$= 4x + 8 + 4x + 6x$

$= 14x + 8$

103. Area of region $= \left(6 \cdot \frac{3}{2}x\right) + \left(6 \cdot \frac{9}{2}x\right)$ or $6 \cdot \left[\frac{3}{2}x + \frac{9}{2}x\right]$

$= 9x + 27x$ or $6\left[\frac{12}{2}x\right]$

$= 36x$ or $36x$

105. Area $= 12(x + 6) - 7x$

$= 12x + 72 - 7x$

$= 5x + 72$

107. (a) *Verbal model:*

Per capita consumption of all beverage milks other than whole	=	Per capita consumption of all beverage milks	−	Per capita consumption of whole milk

Equation:

$$y = (231.06 + 0.009t - 0.095t^2) - (171.17 - 11.415t + 0.325t^2)$$

$$y = 231.06 + 0.009t - 0.095t^2 + (-171.17 + 11.415t - 0.325t^2)$$

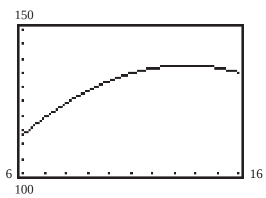
$$= (231.06 - 171.17) + (0.009t + 11.415t) + (-0.095t^2 - 0.325t^2)$$

$$= 59.89 + 11.424t - 0.42t^2$$

$$= -0.42t^2 + 11.424t + 59.89$$

(b) *Keystrokes:*

[Y=] 59.89 [+] 11.4141 [X,T,θ] [−] .42 [X,T,θ] [x^2] [GRAPH]



No, this model was increasing over the interval $6 \le t \le 13.6$.

109. The degree of the term ax^k is k. The term of highest degree in a polynomial has the same degree as the polynomial.

111. $8x^2 - 3x^2 = (8 - 3)x^2 = 5x^2$

113. No, not every trinomial is a second-degree polynomial. For example, $x^3 + 2x + 3$ is a trinomial of third-degree.

Section 3.2 Multiplying Polynomials

1. $t^3 \cdot t^4 = (t \cdot t \cdot t)(t \cdot t \cdot t \cdot t) = t^{3+4} = t^7$

3. $(-5x)^5 = -5x \cdot -5x \cdot -5x \cdot -5x \cdot -5x$

$= -5 \cdot -5 \cdot -5 \cdot -5 \cdot -5 \cdot x \cdot x \cdot x \cdot x \cdot x$

$= (-5)^5x^5 = -3125x^5$

5. $(u^4)^2 = u^4 \cdot u^4$

$= u^{4+4}$

$= u^8$

7. $\dfrac{x^6}{x^4} = \dfrac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = x^{6-4} = x^2$

9. $\left(\dfrac{y}{5}\right)^4 = \dfrac{y}{5} \cdot \dfrac{y}{5} \cdot \dfrac{y}{5} \cdot \dfrac{y}{5} = \dfrac{y \cdot y \cdot y \cdot y}{5 \cdot 5 \cdot 5 \cdot 5} = \dfrac{y^4}{5^4} = \dfrac{y^4}{625}$