

81. —CONTINUED—

Depth of water = $9 - y$

(a) Depth = $9 - y$

$9 = 9 - y$

$0 = y$

$x - 8(0) = 0$

$x = 0$

(b) Depth = $9 - y$

$8 = 9 - y$

$-1 = -y$

$1 = y$

$x - 8(1) = 0$

$x = 8$

(c) Depth = $9 - y$

$7 = 9 - y$

$-2 = -y$

$2 = y$

$x - 8(2) = 0$

$x = 16$

(d) Depth = $9 - y$

$6 = 9 - y$

$-3 = -y$

$3 = y$

$x - 8(3) = 0$

$x = 24$

(e) Depth = $9 - y$

$5 = 9 - y$

$-4 = -y$

$4 = y$

$x - 8(4) = 0$

$x = 32$

(f) Depth = $9 - y$

$4 = 9 - y$

$-5 = -y$

$5 = y$

$x - 8(5) = 0$

$x = 40$

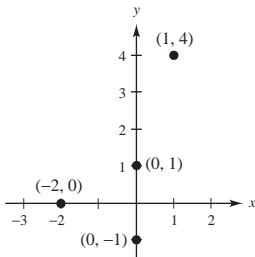
83. Yes. When different pairs of points are selected, the change in y and the change in x are the lengths of the sides of similar triangles. Corresponding sides of similar triangles are proportional.

85. In the equation $y = 3x + 5$, 3 is the slope and 5 is the y -intercept.

Section 2.5 Relations and Functions

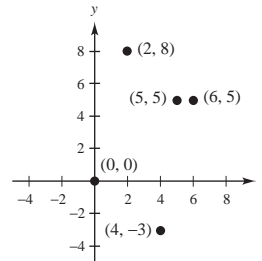
1. Domain = $\{-2, 0, 1\}$

Range = $\{-1, 0, 1, 4\}$



3. Domain = $\{0, 2, 4, 5, 6\}$

Range = $\{-3, 0, 5, 8\}$



5. $(3, 150), (2, 100), (8, 400), (6, 300), (\frac{1}{2}, 25)$

7. $(1, 1), (2, 8), (3, 27), (4, 64), (5, 125), (6, 216), (7, 343)$

9. (1995, Atlanta Braves), (1996, New York Yankees), (1997, Florida Marlins), (1998, New York Yankees)

11. No, this relation is not a function because -1 in the domain is paired to 2 numbers (6 and 7) in the range.

13. Yes, this relation is a function as each number in the domain is paired with exactly one number in the range.

15. No, this relation is not a function as 0 in the domain is paired with 2 numbers in the range (5 and 9).

17. No, this relation is not a function because both CBS and ABC in the domain are each paired to 3 different TV shows in the range.
19. Yes, this relation is a function as each number in the domain is paired with exactly one number in the range.

21. No, this relation is not a function as the 4 and the 7 in the domain are each paired with 2 different numbers in the range.

23. (a) Yes, this relation is a function as each number in the domain is paired with exactly one number in the range.
 (b) No, this relation is not a function as the 1 in the domain is paired with 2 different numbers in the range.
 (c) Yes, this relation is a function as each number in the domain is paired with exactly one number in the range.
 (d) No, this relation is not a function as each number in the domain is not paired with a number.

25. $x^2 + y^2 = 25$

$$\begin{array}{l} 0^2 + 5^2 \stackrel{?}{=} 25 \\ 25 = 25 \end{array} \quad \begin{array}{l} 0^2 + (-5)^2 \stackrel{?}{=} 25 \\ 25 = 25 \end{array}$$

Both (0, 5) and (0, -5) are solutions of $x^2 + y^2 = 25$ which implies y is not a function of x .

27. $|y| = x + 2$

$$\begin{array}{l} |3| \stackrel{?}{=} 1 + 2 \\ 3 = 3 \end{array} \quad \begin{array}{l} |-3| \stackrel{?}{=} 1 + 2 \\ 3 = 3 \end{array}$$

Both (1, 3) and (1, -3) are solutions of $|y| = x + 2$ which implies y is not a function of x .

29. $y = 10x + 12$ represents y as a function of x because there is one value of y associated with one value of x .

31. $3x + 7y - 2 = 0$ represents y as a function of x because there is one value of y associated with one value of x .

33. $y = x(x - 10)$ represents y as a function of x because there is one value of y associated with one value of x .

35. $f(x) = 3x + 5$

- (a) $f(2) = 3(2) + 5 = 11$
 (b) $f(-2) = 3(-2) + 5 = -1$
 (c) $f(k) = 3(k) + 5 = 3k + 5$
 (d) $f(k + 1) = 3(k + 1) + 5 = 3k + 3 + 5 = 3k + 8$

37. $f(x) = 3 - x^2$

- (a) $f(0) = 3 - 0^2 = 3$
 (b) $f(-3) = 3 - (-3)^2 = 3 - 9 = -6$
 (c) $f(m) = 3 - m^2$
 (d) $f(2t) = 3 - (2t)^2 = 3 - 4t^2$

39. $f(x) = \frac{x}{x + 2}$

- (a) $f(3) = \frac{3}{3 + 2} = \frac{3}{5}$
 (b) $f(-4) = \frac{-4}{-4 + 2} = \frac{-4}{-2} = 2$
 (c) $f(s) = \frac{s}{s + 2}$
 (d) $f(s - 2) = \frac{s - 2}{(s - 2) + 2} = \frac{s - 2}{s}$

41. $f(x) = 12x - 7$

(a) $f(3) = 12(3) - 7 = 29$

(b) $f\left(\frac{3}{2}\right) = 12\left(\frac{3}{2}\right) - 7 = 11$

(c) $f(a) + f(1) = [12(a) - 7] + [12(1) - 7] = 12a - 7 + 12 - 7 = 12a - 2$

(d) $f(a + 1) = 12(a + 1) - 7 = 12a + 12 - 7 = 12a + 5$

43. $g(x) = 2 - 4x + x^2$

(a) $g(4) = 2 - 4(4) + 4^2 = 2 - 16 + 16 = 2$

(b) $g(0) = 2 - 4(0) + 0^2 = 2$

(c) $g(2y) = 2 - 4(2y) + (2y)^2 = 2 - 8y + 4y^2$

(d) $g(4) + g(6) = [2 - 4(4) + 4^2] + [2 - 4(6) + 6^2] = (2 - 16 + 16) + (2 - 24 + 36) = 2 + 14 = 16$

45. $f(x) = \sqrt{x + 5}$

(a) $f(-1) = \sqrt{-1 + 5} = 2$

(b) $f(4) = \sqrt{4 + 5} = 3$

(c) $f(z - 5) = \sqrt{z - 5 + 5} = \sqrt{z}$

(d) $f(5z) = \sqrt{5z + 5}$

47. $g(x) = 8 - |x - 4|$

(a) $g(0) = 8 - |0 - 4| = 8 - 4 = 4$

(b) $g(8) = 8 - |8 - 4| = 8 - 4 = 4$

(c) $g(16) - g(-1) = (8 - |16 - 4|) - (8 - |-1 - 4|) = (8 - 12) - (8 - 5) = -4 - 3 = -7$

(d) $g(x - 2) = 8 - |x - 2 - 4| = 8 - |x - 6|$

49. $f(x) = \frac{3x}{x - 5}$

(a) $f(0) = \frac{3(0)}{0 - 5} = 0$

(b) $f\left(\frac{5}{3}\right) = \frac{3\left(\frac{5}{3}\right)}{\frac{5}{3} - 5} \cdot \frac{3}{3} = \frac{15}{5 - 15} = \frac{15}{-10} = -\frac{3}{2}$

(c) $f(2) - f(-1) = \left[\frac{3(2)}{2 - 5}\right] - \left[\frac{3(-1)}{-1 - 5}\right] = \frac{6}{-3} = \frac{-3}{-6} = -2 - \frac{1}{2} = \frac{-5}{2}$

(d) $f(x + 4) = \frac{3(x + 4)}{x + 4 - 5} = \frac{3x + 12}{x - 1}$

51. $f(x) = \begin{cases} x + 8, & \text{if } x < 0 \\ 10 - 2x, & \text{if } x \geq 0 \end{cases}$

(a) $f(4) = 10 - 2(4) = 10 - 8 = 2$

(b) $f(-10) = -10 + 8 = -2$

(c) $f(0) = 10 - 2(0) = 10$

(d) $f(6) - f(-2) = [10 - 2(6)] - [-2 + 8] = 10 - 12 - 6 = -8$

$$53. h(x) = \begin{cases} 4 - x^2, & \text{if } x \leq 2 \\ x - 2, & \text{if } x > 2 \end{cases}$$

$$(a) h(2) = 4 - 2^2 = 0$$

$$(b) h\left(-\frac{3}{2}\right) = 4 - \left(-\frac{3}{2}\right)^2 = 4 - \frac{9}{4} = \frac{16}{4} - \frac{9}{4} = \frac{7}{4}$$

$$(c) h(5) = 5 - 2 = 3$$

$$(d) h(-3) + h(7) = [4 - (-3)^2] + [7 - 2] = 4 - 9 + 5 = 0$$

$$55. f(x) = 2x + 5$$

$$(a) \frac{f(x+2) - f(2)}{x} = \frac{[2(x+2) + 5] - [2(2) + 5]}{x} = \frac{2x + 4 + 5 - 4 - 5}{x} = \frac{2x}{x} = 2$$

$$(b) \frac{f(x-3) - f(3)}{x} = \frac{[2(x-3) + 5] - [2(3) + 5]}{x} = \frac{2x - 6 + 5 - 6 - 5}{x} = \frac{2x - 12}{x}$$

57. Domain of $f(x) = 5 - 2x$ is all real numbers x .

59. Domain of $f(x) = \frac{2x}{x-3}$ is all real numbers x such that $x \neq 3$ because $x - 3 \neq 0$ means $x \neq 3$.

61. Domain of $f(t) = \frac{t+3}{t(t+2)}$ is all real numbers t such that $t \neq 0, -2$ because $t(t+2) \neq 0$ means $t \neq 0$ and $t \neq -2$.

63. Domain of $g(x) = \sqrt{x+4}$ is all real numbers x such that $x \geq -4$ because $x + 4 \geq 0$ means $x \geq -4$.

65. Domain of $f(x) = \sqrt{2x-1}$ is all real numbers x such that $x \geq \frac{1}{2}$ because $2x - 1 \geq 0$ means $x \geq \frac{1}{2}$.

67. Domain of $f(t) = |t - 4|$ is all real numbers t .

69. Domain = $\{0, 2, 4, 6\}$

Range = $\{0, 1, 8, 27\}$

71. Domain = $\{-3, -1, 4, 10\}$

Range = $\left\{-\frac{17}{2}, -\frac{5}{2}, 2, 11\right\}$

73. Domain = $r > 0$

Range = $C > 0$

75. Domain = $r > 0$

Range = $A > 0$

77. Verbal model: $\boxed{\text{Perimeter}} = 4 \cdot \boxed{\text{Length of side}}$

Labels: Perimeter = $P(x)$

Length of side = x

Function: $P(x) = 4x$

79. Verbal model: $\boxed{\text{Volume}} = \boxed{\text{Length of side}}^3$

Labels: Volume = $V(x)$

Length of side = x

Function: $V(x) = x^3$

81. Verbal model: $\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$

Labels: Distance = $d(t)$

Rate = 230

Time = t

Function: $d(t) = 230t$

83. Verbal model: $\boxed{\text{Volume}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}} \cdot \boxed{\text{Height}}$

Labels: Volume = $V(x)$

Length = $(24 - 2x)$

Width = $(24 - 2x)$

Height = x

Function: $V(x) = x(24 - 2x)^2$ or $4x(12 - x)^2$

85. Verbal model: $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Labels: Area = $A(x)$

Length = $(32 - x)$

Width = $(32 - x)$

Function: $A(x) = (32 - x)(32 - x)$

$A(x) = (32 - x)^2$

87. $S(L) = \frac{128,160}{L}$

(a) $S(12) = \frac{128,160}{12} = 10,680$ pounds

(b) $S(16) = \frac{128,160}{16} = 8010$ pounds

89. Yes to both questions. For each year there is associated one public school enrollment and one private school enrollment.

91. (g) $y(x) = 15,900 - 1900x$

$y(7) = 15,900 - 1900(7)$

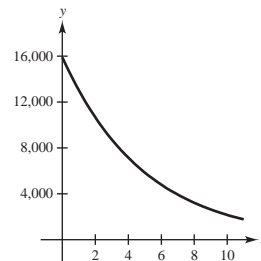
$= 15,900 - 13,300$

$= \$2600$

(i) Domain: all real numbers x such that $0 < x \leq 8.37$

Range: all real numbers y such that $0 < y \leq 15,900$

(h) Straight-line depreciation might not be a fair model for automobile depreciation because the car depreciates more slowly as the car ages.



93. (a) This is not a correct mathematical use of the word function.

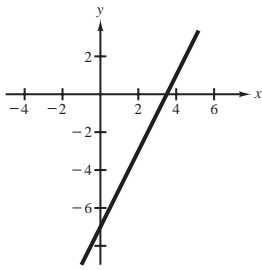
(b) This is a correct mathematical use of the word function.

95. No, every relation is not a function because some relations have more than one y value paired with each x value. For example, $\{(4, 3), (4, -2)\}$ is a relation, but not a function.

97. You can name the function (f, g , etc.). That is convenient when there is more than one function used in solving a problem. The values of the independent and the dependent variables are easily seen in function notation.

Section 2.6 Graphs of Functions

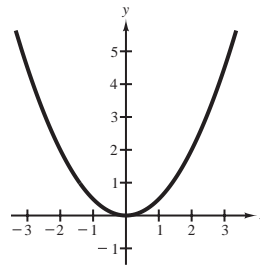
1.



Domain: $-\infty < x < \infty$

Range: $-\infty < y < \infty$

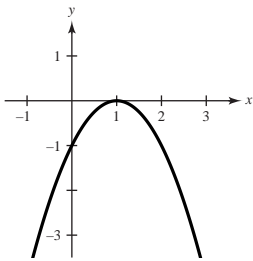
3.



Domain: $-\infty < x < \infty$

Range: $[0, \infty)$ or $0 \leq y < \infty$

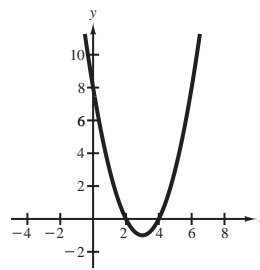
5.



Domain: $-\infty < x < \infty$

Range: $(-\infty, 0]$ or $-\infty < y \leq 0$

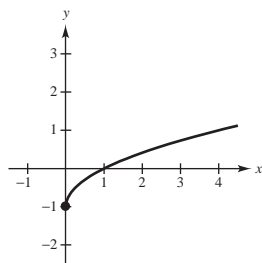
7.



Domain: $-\infty < x < \infty$

Range: $[-1, \infty)$ or $-1 \leq y < \infty$

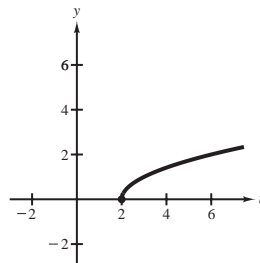
9.



Domain: $0 \leq x < \infty$

Range: $-1 \leq y < \infty$

11.



Domain: $[2, \infty)$ or $2 \leq t < \infty$

Range: $[0, \infty)$ or $0 \leq y < \infty$