36. Rework Exercise 35 assuming that each one-page newspaper ad costs $30,000.

In Exercises 37 and 38, use the following information. A computer company has two assembly plants, Plant A and Plant B, and two distribution outlets, Outlet I and Outlet II. Plant A can assemble 5000 computers in a year and Plant B can assemble 4000 computers in a year. Outlet I must have 3000 computers per year and Outlet II must have 5000 computers per year. The transportation costs from each plant to each outlet are indicated in the given table. Find the shipping schedule that will produce the minimum cost. What is the minimum cost?

37.

	Outlet I	*Outlet II*
Plant A	$4	$5
Plant B	$5	$6

38.

	Outlet I	*Outlet II*
Plant A	$4	$5
Plant B	$6	$4

CHAPTER 9 ❑ REVIEW EXERCISES

In Exercises 1–6, sketch a graph of the solution of the system of inequalities.

1. $x + 2y \le 160$, $3x + y \le 180$, $x \ge 0$, $y \ge 0$

2. $2x + 3y \le 24$, $2x + y \le 16$, $x \ge 0$, $y \ge 0$

3. $3x + 2y \ge 24$, $x + 2y \ge 12$, $2 \le x \le 15$, $y \le 15$

4. $2x + y \ge 16$, $x + 3y \ge 18$, $0 \le x \le 25$, $0 \le y \le 15$

5. $2x - 3y \ge 0$, $2x - y \le 8$, $y \ge 0$

6. $x - y \le 10$, $x \ge 0$, $y \ge 0$

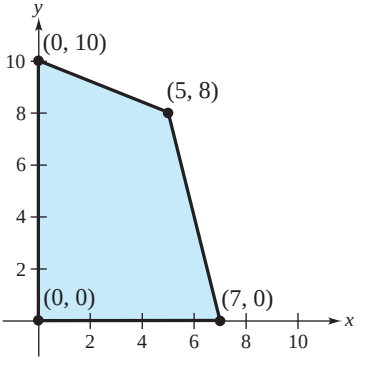
In Exercises 7 and 8, determine a system of inequalities that models the given description, and sketch a graph of the solution of the system.

7. A Pennsylvania fruit grower has 1500 bushels of apples that are to be divided between markets in Harrisburg and Philadelphia. These two markets need at least 400 bushels and 600 bushels, respectively.

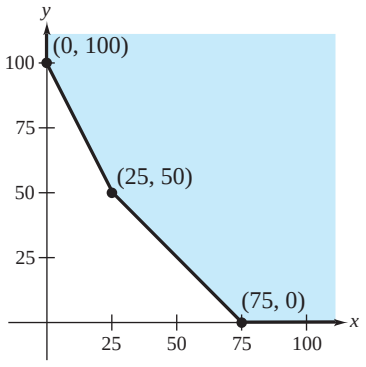
8. A warehouse operator has 24,000 square meters of floor space in which to store two products. Each unit of product I requires 20 square meters of floor space and costs $12 per day to store. Each unit of product II requires 30 square meters of floor space and costs $8 per day to store. The total storage cost per day cannot exceed $12,400.

In Exercises 9–14, find the minimum and/or maximum values of the given objective function by the graphical method.

9. Maximize: $z = 3x + 4y$



10. Minimize: $z = 10x + 7y$



11. Objective function:
$z = x + 3y$
Constraints:
$x \geq 0$
$y \geq 0$
$x \leq 5$
$x + y \geq 3$
$x - y \leq 3$
$-x + y \leq 3$

12. Objective function:
$z = 4x - y$
Constraints:
$x \geq 0$
$y \geq 0$
$x \leq 6$
$x + y \geq 2$
$x \geq y$
$3x - y \leq 12$

13. Objective function:
$z = 3x - y$
Constraints:
$x \geq 0$
$y \geq 0$
$x \leq 3y$
$-x + 2y \leq 12$
$4x + 3y \leq 40$
$x + 2y \leq 15$

14. Objective function:
$z = x - 2y$
Constraints:
$x \geq 0$
$y \geq 0$
$x + y \leq 12$
$5x + y \leq 36$
$5x - 2y \geq 4$
$2x + 5y \geq 19$

In Exercises 15–22, use the simplex method to maximize the given objective function, subject to the given constraints.

15. Objective function:
$z = x_1 + 2x_2$
Constraints:
$2x_1 + x_2 \leq 31$
$x_1 + 4x_2 \leq 40$
$x_1, x_2 \geq 0$

16. Objective function:
$z = 5x_1 + 4x_2$
Constraints:
$x_1 - x_2 \leq 22$
$x_1 + 2x_2 \leq 43$
$x_1, x_2 \geq 0$

17. Objective function:
$z = x_1 + 2x_2 + x_3$
Constraints:
$2x_1 + 2x_2 + x_3 \leq 20$
$x_1 + x_2 - 2x_3 \leq 23$
$-2x_1 + x_2 - 2x_3 \leq 8$
$x_1, x_2, x_3 \geq 0$

18. Objective function:
$z = 4x_1 + 5x_2 + 6x_3$
Constraints:
$4x_1 + 2x_2 + x_3 \leq 30$
$x_1 + 3x_2 + 2x_3 \leq 54$
$x_1 + x_2 + 2x_3 \leq 24$
$x_1, x_2, x_3 \geq 0$

19. Objective function:
$z = x_1 + x_2$
Constraints:
$3x_1 + x_2 \leq 432$
$x_1 + 4x_2 \leq 628$
$x_1, x_2 \geq 0$

20. Objective function:
$z = 6x_1 + 8x_2$
Constraints:
$20x_1 + 40x_2 \leq 200$
$30x_1 + 42x_2 \leq 228$
$x_1, x_2 \geq 0$

21. Objective function:
$z = 3x_1 + 5x_2 + 4x_3$
Constraints:
$6x_1 - 2x_2 + 3x_3 \leq 24$
$3x_1 - 3x_2 + 9x_3 \leq 33$
$-2x_1 + x_2 - 2x_3 \leq 25$
$x_1, x_2, x_3 \geq 0$

22. Objective function:
$z = 2x_1 + 5x_2 - x_3$
Constraints:
$-x_1 + 3x_2 + 2x_3 \leq 92$
$-2x_1 + 2x_2 + 12x_3 \leq 76$
$3x_1 + x_2 - 6x_3 \leq 24$
$x_1, x_2, x_3 \geq 0$

In Exercises 23 and 24, determine the dual of the given minimization problem.

23. Objective function:
$w = 7x_1 + 3x_2 + x_3$
Constraints:
$x_1 + x_2 + 2x_3 \geq 30$
$3x_1 + 6x_2 + 4x_3 \geq 75$
$x_1, x_2, x_3 \geq 0$

24. Objective function:
$w = 2x_1 + 3x_2 + 4x_3$
Constraints:
$x_1 + 5x_2 + 3x_3 \geq 90$
$2x_1 + x_2 + 3x_3 \geq 60$
$3x_1 + 2x_2 + x_3 \geq 56$
$x_1, x_2, x_3 \geq 0$

In Exercises 25–30, solve the given minimization problem by solving the dual maximization problem with the simplex method.

25. Objective function:
$w = 9x_1 + 15x_2$
Constraints:
$x_1 + 5x_2 \geq 15$
$4x_1 - 10x_2 \geq 0$
$x_1, x_2 \geq 0$

26. Objective function:
$w = 16x_1 + 18x_2$
Constraints:
$2x_1 - 3x_2 \geq 14$
$-4x_1 + 9x_2 \geq 8$
$x_1, x_2 \geq 0$

27. Objective function:

$$w = 24x_1 + 22x_2 + 18x_3$$

Constraints:

$$\begin{aligned} 2x_1 + 2x_2 - 3x_3 &\geq 24 \\ 6x_1 \qquad - 2x_3 &\geq 21 \\ -8x_1 - 4x_2 + 8x_3 &\geq 12 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

28. Objective function:

$$w = 32x_1 + 36x_2 + 4x_3$$

Constraints:

$$\begin{aligned} 4x_1 + 3x_2 - x_3 &\geq 8 \\ -8x_1 + 6x_2 - 6x_3 &\geq 0 \\ -4x_1 \qquad + 9x_3 &\geq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

29. Objective function:

$$w = 16x_1 + 54x_2 + 48x_3$$

Constraints:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &\geq 2 \\ 2x_1 + 7x_2 + 4x_3 &\geq 5 \\ x_1 + 3x_2 + 4x_3 &\geq 1 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

30. Objective function:

$$w = 22x_1 + 27x_2 + 18x_3$$

Constraints:

$$\begin{aligned} -2x_1 - 3x_2 + 6x_3 &\geq 0 \\ -2x_1 + 7x_2 + 3x_3 &\geq 4 \\ 2x_1 + x_2 - 3x_3 &\geq 12 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

In Exercises 31–36, use the simplex method to solve the mixed constraints problem.

31. (Maximize)

Objective function:

$$z = x_1 + 2x_2$$

Constraints:

$$\begin{aligned} -4x_1 + 2x_2 &\leq 26 \\ -3x_1 + x_2 &\geq 12 \\ x_1, x_2 &\geq 0 \end{aligned}$$

32. (Maximize)

Objective function:

$$z = 2x_1 + 3x_2$$

Constraints:

$$\begin{aligned} -x_1 + x_2 &\geq 40 \\ x_2 &\leq 61 \\ x_1, x_2 &\geq 0 \end{aligned}$$

33. (Maximize)

Objective function:

$$z = 2x_1 + x_2 + x_3$$

Constraints:

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 60 \\ -4x_1 + 2x_2 + x_3 &\geq 52 \\ 2x_1 \qquad + x_3 &\geq 40 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

34. (Maximize)

Objective function:

$$z = 3x_1 + 2x_2 + x_3$$

Constraints:

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &\leq 52 \\ x_1 + x_2 + 2x_3 &\geq 24 \\ 2x_2 + x_3 &\leq 52 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

35. (Minimize)

Objective function:

$$z = 9x_1 + 4x_2 + 10x_3$$

Constraints:

$$\begin{aligned} 32x_1 + 16x_2 + 8x_3 &\leq 344 \\ 20x_1 - 40x_2 + 20x_3 &\geq 200 \\ -45x_1 + 15x_2 + 30x_3 &\leq 525 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

36. (Minimize)

Objective function:

$$z = 4x_1 - 2x_2 - x_3$$

Constraints:

$$\begin{aligned} 2x_1 - x_2 - x_3 &\leq 41 \\ x_1 - 2x_2 - x_3 &\geq 10 \\ -x_1 - 7x_2 + 5x_3 &\leq 11 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

37. A tailor has 12 square feet of cotton, 21 square feet of silk, and 11 square feet of wool. A vest requires 1 square foot of cotton, 2 square feet of silk, and 3 square feet of wool. A purse requires 2 square feet of cotton, 1 square foot of silk, and 1 square foot of wool. If the purse sells for \$80 and the vest sells for \$50, how many purses and vests should be made to maximize the tailor's profit? What is the maximum revenue?

38. A carpenter has 400 board feet of plywood, 487 board feet of birch, and 795 board feet of pine. Product A requires 1 board foot of plywood, 2 board feet of birch, and 1 board foot of pine. Product B requires 1 board foot of plywood, 1 board foot of birch, and 3 board feet of pine. Product C requires 2 board feet of plywood, 1 board foot of birch, and 1 board foot of pine. If product A sells for \$22, product B sells for \$42, and product C sells for \$29, what combination of products would yield the maximum gross income?

39. A company owns three mines that have the following daily production (in metric tons).

Mine	*Grade of Ore*		
	High	*Medium*	*Low*
A	1	2	3
B	1	2	2
C	2	1	1

The company needs 60 metric tons of high-grade ore, 48 metric tons of medium-grade ore, and 55 metric tons of low-grade ore. Minimize the cost of meeting this production level if the daily operating costs are \$200 for mine A, \$200 for mine B, and \$100 for mine C.

40. Rework Exercise 39 given the following daily production schedule.

Mine	*Grade of Ore*		
	High	*Medium*	*Low*
A	2	1	2
B	1	1	1
C	4	2	1

The company needs 190 metric tons of high-grade ore, 120 metric tons of medium-grade ore, and 200 tons of low-grade ore. The daily operating costs are \$200 for mine A, \$150 for mine B, and \$125 for mine C.