Chapter 3 Polynomial Functions

Section 3.1

Polynomial function – Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ is called a polynomial function of x with degree n

Constant function – A polynomial function with degree 0. That is, $f(x) = a$, $a \neq 0$.

Linear function – A polynomial function with degree 1. That is, $f(x) = ax + b$, $a \neq 0$.

Quadratic function – Let a, b, and c be real numbers with $a \neq 0$. The function $f(x) = ax^2 + bx + c$ is called a quadratic function.

Parabola – Name for a special type of "U" shaped curve that is the graph of a quadratic function

Axis – A line about which a parabola is symmetric

Vertex – The point where the axis intersects the parabola

Standard form of a quadratic function – The standard form of a quadratic function is $f(x) = a(x - h)^2 + k$, $a \neq 0$.

Section 3.2

Continuous – The graph of a polynomial function has no breaks, holes, or gaps.

Leading Coefficient Test – As x moves without bound to the left or to the right, the graph of the polynomial function $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ eventually rises or falls in the following manner:

1. When n is odd:

a. If the leading coefficient is positive, the graph falls to the left and rises to the right.

b. If the leading coefficient is negative, the graph rises to the left and falls to the right.

2. When n is even:

a. If the leading coefficient is positive, the graph rises to the left and right.

b. If the leading coefficient is negative, the graph falls to the left and right.

Repeated zero – If $(x - a)^k$, $k > 1$ is a factor of a polynomial, then $x = a$ is a repeated zero

Multiplicity – The number of times a zero is repeated

Intermediate Value Theorem – Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

Section 3.3

Long division – A procedure for dividing two polynomials, which is similar to long division in arithmetic

Division Algorithm – If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that $f(x) = d(x)q(x) + r(x)$ where $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $d(x)$.

Improper – A rational expression $f(x)/d(x)$ where the degree of $f(x)$ is greater than the degree of $d(x)$

Proper – A rational expression $r(x)/d(x)$ where the degree of $r(x)$ is less than the degree of $d(x)$

Synthetic division – A shortcut for long division of polynomials when dividing by divisors of the form $x - k$

Remainder Theorem – If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$

Factor Theorem – A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$

Section 3.4

Fundamental Theorem of Algebra – If $f(x)$ is a polynomial of degree n, where $n > 0$, then f has at least one zero in the complex number system

Linear Factorization Theorem – If $f(x)$ is a polynomial of degree n, where $n > 0$, then f has precisely n linear factors $f(x) = a_n(x - c_1)(x - c_2) \ldots (x - c_n)$ where $c_1, c_2, \ldots, c_n$ are complex numbers

Rational Zero Test – If the polynomial $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ has integer coefficients, every rational zero of f has the form

Rational zero = $\dfrac{p}{q}$

where p and q have no common factors other than 1, and p = a factor of the constant term a_0 and q = a factor of the leading coefficient a_n

Conjugates – A pair of complex numbers of the form $a + bi$ and $a - bi$ are complex conjugates of each other
Irreducible over the reals – A quadratic factor with no real zeros
Descartes's Rule of Signs – Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ be a polynomial with real coefficients and $a_0 \neq 0$.

1. The number of positive real zeros of f is either equal to the number of variations in sign of $f(x)$ or less than that number by an even integer.
2. The number of negative real zeros of f is either equal to the number of variations in sign of $f(-x)$ or less than that number by an even integer.

Variation in sign – Two consecutive coefficients have opposite signs
Upper bound – A real number b is an upper bound for the real zeros of f if no real zeros of f are greater than b
Lower bound – A real number b is a lower bound for the real zeros of f if no real zeros of f are less than b

Section 3.5
Vary directly – if $y = kx$ for some nonzero constant k, then y is said to vary directly as x
Directly proportional – If y varies directly as x, then y is directly proportional to x
Constant of variation – A nonzero constant k in the equation $y = kx$
Vary inversely – If x and y are related by an equation of the form $y = k/x$, then y varies inversely as x
Inversely proportional – If y varies inversely as x, then y is directly proportional to x
Vary jointly – If $z = kxy$ for some constant k, then z varies jointly as x and y
Jointly proportional – If z varies jointly as x and y, then z is jointly proportional to x and y
Sum of square differences – The sum of the squares of the differences between actual data values and model values
Least squares regression line – The best fitting linear model with the least sum of square differences