Objective: **Use a differential equation to model and solve an applied problem.**

A falling object encounters air resistance that is proportional to its velocity. If the acceleration due to gravity is –9.8 meters per second, the net change in velocity is $\frac{dv}{dt} = kv - 9.8$. Find the velocity of the object as a function of time if the initial velocity is V_0.

ANSWER:

$$\frac{dv}{dt} = kv - 9.8$$

$$\frac{dv}{kv - 9.8} = dt$$

$$\int \frac{dv}{kv - 9.8} = \int dt$$ Remember: (Use "u" substitution.)

$$\ln|kv - 9.8| = kt + c_2$$
$$e^{\ln|kv-9.8|} = e^{kt+c_2}$$
$$kv - 9.8 = e^{kt+c_2} = C_3 e^{kt}$$
$$V = \frac{1}{k}\left[9.8 + C_3 e^{kt}\right]$$
At $t = 0$, $V_0 = \frac{1}{k}\left[9.8 + C_3\right]$ $*\left(C_3 = kv_0 - 9.8\right)*$

$$V = \frac{1}{k}\left[9.8 + \left(kv_0 - 9.8\right)e^{kt}\right]$$
Note that $k < 0$ since the object is moving downward.