Objective: **Recognize and solve homogeneous differential equations.**

Solve the homogeneous differential equation $y' = \dfrac{\left(x^3 + y^3\right)}{xy^2}$.

ANSWER:

$$y' = \frac{\left(x^3 + y^3\right)}{xy^2}$$
$$xy^2\,dy = \left(x^3 + y^3\right)dx$$
$$y = vx,\ dy = x\,dv + v\,dx$$
$$x(vx)^2\left(x\,dv + v\,dx\right) = \left(x^3 + (vx)^3\right)dx$$
$$x^4v^2\,dv + x^3v^3\,dx = x^3\,dx + v^3x^3\,dx$$
$$xv^2\,dv = dx$$
$$\int v^2\ dv = \int \frac{1}{x}\,dx$$
$$\frac{v^3}{3} = \ln|x| + C$$
$$\left(\frac{y}{x}\right)^3 = 3\ln|x| + C$$
$$y^3 = 3x^3\ln|x| + Cx^3$$