

Objective: Use initial conditions to find particular solutions of differential equations.

Verify that the general solution $3x^2 + 2y^2 = C$ satisfies the differential equation $3x + 2yy' = 0$. Then find the particular solution that satisfies the initial condition $y = 3$ when $x = 1$.

ANSWER:

$$3x^2 + 2y^2 + C$$

Find the derivative using implicit differentiation.

$$6x + 4yy' = 0$$

$$y' = \frac{-6x}{4y}$$
$$= \frac{-3x}{2y}$$

Now we must find y .

$$3x^2 + 2y^2 + C$$

$$2y^2 = C - 3x^2$$

$$y^2 = \frac{C - 3x^2}{2}$$

$$y = \sqrt{\frac{C - 3x^2}{2}}$$

so, $3x + 2yy' = 0$

$$3x + 2\sqrt{\frac{C - 3x^2}{2}} \left(-\frac{3x}{2y} \right) = 0$$

$$3x - \frac{6x}{2y} \sqrt{\frac{C - 3x^2}{2}} = 0$$

$$3x - \frac{6x}{2\sqrt{\frac{C - 3x^2}{2}}} \cdot \sqrt{\frac{C - 3x^2}{2}} = 0$$

$$3x - \frac{6x}{2} = 0$$

$$3x - 3x = 0$$

$0 = 0$ Therefore, the general solution satisfies the differential equation.

Next, we must find the particular solution that satisfies the initial condition $y = 3$ when $x = 1$.

$$3x^2 + 2y^2 = C$$

$$3(1)^2 + 2(3)^2 = C$$

$$3 + 18 = C$$

$$21 = C$$

So, the particular solution is $3x^2 + 2y^2 = 21$