

Objective: Use the separation of variables to solve a simple differential equation.

Solve the differential equation $xy + y' = 100x$.

ANSWER:

$$xy + y' = 100x$$

$$y' = 100x - xy$$

$$y' = x(100 - y)$$

$$\frac{y'}{100 - y} = x$$

$$\int \frac{y'}{100 - y} dx = \int x dx$$

$$\int \frac{y'}{100 - y} dy = \int x dx \quad \text{Remember: } (dy = y' dx)$$

Let $u = 100 - y$

$$\frac{du}{dy} = -1$$

$$du = -1 dy$$

$$\frac{du}{-1} = dy$$

$$\text{so, } - \int \frac{1}{u} du = \int x dx$$

$$- \ln u = \frac{x^2}{2} + C$$

$$- \ln(100 - y) = \frac{x^2}{2} + C$$

$$\ln(100 - y) = -\frac{x^2}{2} - C$$

$$e^{\ln(100-y)} = e^{\left(-\frac{x^2}{2}\right) - C}$$

$$100 - y = e^{\left(-\frac{x^2}{2}\right) - C}$$

$$-y = e^{\left(-\frac{x^2}{2}\right) - C} - 100$$

$$-y = e^{-c} e^{\left(\frac{-x^2}{2}\right)} - 100 \quad ** (e^{-c} \text{ is a constant so it can be written as } C.) **$$

$$y = -C e^{\left(\frac{-x^2}{2}\right)} + 100$$