

Objective: Use the Log Rule for Integration to integrate a rational function.

Evaluate the integral $\int_1^e \frac{\ln x}{x} dx$.

ANSWER:

$$\int_1^e \frac{\ln x}{x} dx$$

Let $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx$$

So, $\int_1^e \frac{\ln x}{x} dx = \int_0^1 u du$ (when $x = 1$, $u = 0$ and when $x = e$, $u = 1$)

$$\begin{aligned} &= \left[\frac{u^2}{2} \right]_0^1 \\ &= \frac{1^2}{2} - \frac{0^2}{2} \\ &= \frac{1}{2} - 0 \\ &= \frac{1}{2} \end{aligned}$$