Objective: **Find the derivatives of functions involving the natural logarithmic function.**

Find the derivative of the function $f(x) = x\sqrt{\ln x}$.

ANSWER:

$f(x) = x\sqrt{\ln x} = x(\ln x)^{\frac{1}{2}}$

The chain rule must be applied:

$$f'(x) = x\left(\frac{1}{2}(\ln x)^{-\frac{1}{2}}\left(\frac{1}{x}\right)\right) + (\ln x)^{\frac{1}{2}}(1)$$

$$= \frac{1}{2(\ln x)^{\frac{1}{2}}} + (\ln x)^{\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{\ln x}} + \sqrt{\ln x}$$

$$= \frac{1}{2\sqrt{\ln x}} + \frac{2\sqrt{\ln x}\sqrt{\ln x}}{2\sqrt{\ln x}}$$

$$= \frac{1}{2\sqrt{\ln x}} + \frac{2\left(\sqrt{\ln x}\right)^2}{2\sqrt{\ln x}}$$

$$= \frac{1 + 2\ln x}{2\sqrt{\ln x}}$$