

**Objective: Differentiate and integrate functions involving inverse hyperbolic functions.**

a.) Find the derivative of the function  $y = x \tanh^{-1} 2x$ .

b.) Find the indefinite integral of  $\int \frac{1}{x\sqrt{4-9x^2}} dx$ .

**ANSWER:**

a.)  $y = x \tanh^{-1} 2x$

$$y' = (1) \tanh^{-1} 2x + x \left( \frac{2}{1-(2x)^2} \right)$$

$$y' = \tanh^{-1} 2x + \left( \frac{2x}{1-4x^2} \right)$$

b.)  $\int \frac{1}{x\sqrt{4-9x^2}} dx$

Let  $a = 2$  and  $u = 3x$

$$\begin{aligned} \int \frac{1}{x\sqrt{4-9x^2}} dx &= \int \frac{3}{(3x)\sqrt{4-9x^2}} dx \\ &= -\frac{1}{2} \ln \frac{2 + \sqrt{4-9x^2}}{|3x|} + C \end{aligned}$$

**Remember:**  $\int \frac{1}{u\sqrt{a^2-u^2}} du$

**Remember:**  $-\frac{1}{a} \ln \frac{a + \sqrt{a^2-u^2}}{|u|} + C$